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The methodology used to simulate the steelmaking process

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Abstract: Because of the research that has been done over the past three decades, continuous casting is now a complex and cutting-edge technique. This study has referenced the critical numerical modeling approach of the continuous casting process. The current work explains how molten steel flows, transfers heat, solidifies, forms the shell through solidification and coupling, and more. The continuous casting process is currently a trusted industrial method for the manufacture of steel. Numerous complex processes involving fluid flow, heat transport, and structural deformation are a part of the continuous casting process. Since metallurgical techniques have recently advanced, the continuous casting process has taken over as the primary way to make steel. Steel producers are always looking for innovative, more productive manufacturing techniques in order to achieve efficient and effective output.

Keywords: steelmaking, continuous casting, numerical modeling,

1. Introduction

Since metallurgical techniques have recently advanced, the continuous casting process has taken over as the primary way to make steel[1][2]. The advantages of continuous projecting, which include cost savings, high efficiency, and better quality, serve as the justification for this [3]–[28]. Numerous complex processes involving fluid flow, heat transport, and structural deformation are a part of the continuous casting process. The crucial component and method of continuous casting have been meticulously modeled and explored in references [3]. Steel producers are always looking for innovative, more productive manufacturing techniques in order to achieve efficient and effective output. The practice of optimizing through numerical modeling is one such technique that has grown in popularity. It describes the formation of the shell by solidification and the flow of molten steel [29]. Additionally, a detailed explanation of strand distortion caused by thermomechanical forces, bulging, bending, and crack prediction has been provided. There are a lot of potent pre-coded solvers on the market right now. In order to produce a high-quality product, numerical simulation of the thermo-mechanical behavior of the continuous casting process is crucial [4], [5], [15], [22]–[28]. The research work done in the last three[13], [20], [29]–[34] decades has made continuous casting an advanced and sophisticated technology[3], [35]–[37]. Physical water models that take into account the viscosity of water equivalent to steel can simulate the flow of molten steel in the mold region of continuous casting processes [38]–[42]. The highly non-linear constitutive laws of structure, the incorporation of latent heat, the presence of three different material states (liquid, mushy, and solid), temperature-dependent material properties, the irregular contact between the mold surface and solidified strand, and the coupling of the heat transfer and structure models with appropriate continuum mechanisms and boundary conditions are just a few of the challenges that must be overcome in this part of the simulation [43]. Reynold's Averaged Navier- Stokes (RANS) method has been widely adopted for turbulence modeling. It has been reported that the RANS model is highly accurate in predicting steady-state flow patterns [24]. Several research works have been done on molten steel flow, heat transfer and solidification in mold[40]–[42], [44]–[46]. These studies have been established and validated with industrial trials[6], [35], [47]–[50]. From all previous studies, it is well established that numerical models efficiently and accurately predict the fluid flow and mechanical behavior of mold and strand, respectively [38] [39]. The most adopted technique for simulating the solidification of continuous casting is the enthalpy-porosity approach [12], [14], [16], [17], [22], [24], [30], [51]–[55]. This method is based on a component called a liquid fraction. Numerous scholars have looked at this strategy, although the majority of them have only done 2D modeling. Although conduction, convection, and radiation are the three mechanisms of heat transport researched in the mold, their effects on the finished product and the potential for reducing any negative effects must be investigated. In order to present up-to-date knowledge on the numerical modeling of continuous casting processes, we examined and evaluated the literature.

2. Heat transport and solidification equations

The fundamental requirement of the continuous casting process is to solidify the strand to achieve plant set quality standards [3], [37], [56]–[64] Generalized heat transfer equation (3-dimension) can be written in the most suitable format from the above equations in the following manner[65] [13], [30], [31], [33], [52], [66]–[79];

$$\rho C\left(\frac{\partial T}{\partial t}+V_x\frac{\partial T}{\partial t}+V_y\frac{\partial T}{\partial t}+V_z\frac{\partial T}{\partial t}\right)=\ddot{q}+\frac{\partial}{\partial x}\left(K_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right) \quad 1$$

In 2005, Louhenkilpi et al. proposed a three-dimensional transient formulation for temperature distribution over the mold wall. [80];

$$\rho\frac{\partial H}{\partial t}+v\frac{\partial H}{\partial z}=\frac{\partial}{\partial x}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial z}\right) \quad 2$$

In a similar work, Zhao et al. (2005) [81] modeled energy equation along with the Navier-Stokes equation.

$$\frac{\partial\bar{T}}{\partial t}+\frac{\partial(\bar{u}_i\bar{T})}{\partial x_i}=\frac{k}{\rho_0 C_P}\frac{\partial}{\partial x_i}\left(\frac{\partial\bar{T}}{\partial x_i}\right)+\frac{\partial Q_{Ti}}{\partial x_i} \quad 3$$

$$Q_{Ti}=\bar{T}\bar{u}_i-\overline{Tu_i} \quad 4$$

$$\frac{\partial Q_{Ti}}{\partial x_i}=\frac{\mu_T}{\mathrm{Pr}_T}\frac{\partial}{\partial x_i}\frac{\partial\bar{T}}{\partial x_i} \quad 5$$

In 2011, Sowa and Bokota [82] proposed a heat flow model based on the Fourier-Kirchhoff system of equations.

$$\rho c\left(\frac{\partial T(\mathbf{x},t)}{\partial t}+\nabla T\cdot\mathbf{v}\right)=\nabla\cdot(\lambda\nabla T)+\dot{Q} \quad 6$$

Sowa and Bakota et al. [82] modified the above equation which includes effective specific heat (Ceff) term which is a function of the temperature of the material.

$$\nabla.(\lambda\Delta T)-C_{ef}\frac{\partial T}{\partial t}-C_{ef}\nabla T.V=0 \quad 7$$

$$C_{ef}(T) = \rho_{LS}c_{LS} + \rho_S L/(T_L - T_S) \tag{8}$$

In 2011, Hadata et al., [45] proposed a steady Fourier-Kirchhoff model for heat flow with some assumptions.

$$q_v = Q_s \frac{dV_S}{d\tau} \tag{9}$$

In a study in 1993 S.E.Chidiac et.at.,[64]used enthalpy approach for heat transfer in multi-dimensional problem with following equation.

$$\rho \frac{\partial H}{\partial t} = \nabla \cdot (K \nabla T) + Q \tag{10}$$

where ρ indicates density, H indicates enthalpy, K indicates Thermal conductivity, Q indicates heat generation rate for unit volume, T indicates temperature and t time. Enthalpy is nothing but the summation of sensible & latent heat and can be expressed as:

$$H = \int_{T_r}^{T} c dT + f(T). L \tag{11}$$

where c, $f(T)$ and L are specific heat liquid fraction and latent heat. For phase change study two methods are clubbed together with the above-stated formulation for accuracy and efficiency. Dirichlet & Cauchy boundary conditions are used to solve above equations. The study carried in 2003, B. wiwanapataphee et. al., [63] for simulating phase change cause of heat transfer single domain enthalpy method is adopted. Where enthalpy is the summation of latent heat (H) & sensible heat (h).

$$H=h+\Delta H \tag{12}$$

Latent heat h can be given by

$$\mathrm{H}=f(T)L, \tag{13}$$

Where L denoted Latent Heat of Steel L and $f(T)$ indicates localized liquid fraction where value one represents complete Liquids state and zero represents the complete solid-state. The liquid fraction is nonlinear for simplification of the model it is assumed linear.

$$f(T) = \begin{cases} 0, & T \leqslant T_S \\ \frac{T - T_S}{T_\mathrm{L} - T_\mathrm{S}}, & T_\mathrm{S} < T < T_\mathrm{L}, \\ 1, & T \geqslant T_\mathrm{L}, \text{ (XX)} \end{cases} \tag{14}$$

wherein T_L indicates melting temperature and T_S Solidification temperature

For region where phase change occurs conservation of energy principle. Combining this equation with enthalpy gives,

$$\rho c \left(\frac{\partial T}{\partial t} + u_j T_{,}\right) = \left(k_0 T_j\right)_j - S_T \tag{15}$$

Ivanova (2013) [83] formulated extensive mathematical modeling on predicting phase-dependent boundary conditions.

$$\frac{\partial T}{\partial r} + v(t) \cdot \frac{\partial T}{\partial z} = \frac{1}{c(T)\rho(T)} \times \\ \times \left\{ \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial \partial_z}\left[\lambda(T)\frac{\partial T}{\partial_z}\right] \right\} \quad 16$$

The position of the unknown phase boundary is specified by the equality condition of the temperatures and the Stefan condition for the two-dimensional case:

$$T = T(\tau, x, z)|_{x=\xi\_(\tau,z)=} T(\tau, x, z)|_{x=\xi_+(\tau,z)=T_{cr}} \quad 17$$

$$\lambda(T)\ \frac{\partial T}{\partial \bar{n}}|\xi_+ - \ \lambda(T)\frac{\partial T}{\partial \bar{n}}|\xi_- = \ \mu\rho\ (T_{kp})\ \left(\frac{d\xi}{d\tau} + v(\tau)\frac{d\xi}{dz}\right) \quad 18$$

where ξ is the phase boundary $x = \xi(\tau, z), \bar{n}$ is a normal to the phase boundary, $\left.\frac{\partial T}{\partial \pi}\right|_{\xi+/-}$ is the left-right limit of the temperature derivative in the normal direction. μ is the latent the heat of crystallization. T_{cr} is the crystallization temperature (the average temperature from the liquidus-solidus interval).

In 2014, Zhang et al [84] investigated a steady-state two-dimensional numerical model based on the assumption of heat transfer.

$$\rho = (1 - f_{\mathrm{S}})\rho_{\mathrm{L}} + f_{\mathrm{S}}\left(f_\delta \rho_\delta + f_\gamma \rho_\gamma\right) \quad 19$$

$$\lambda = (1 - f_{\mathrm{S}})\lambda_{\mathrm{L}} + f_{\mathrm{S}}\left(f_\delta \lambda_\delta + f_\gamma \lambda_\gamma\right) \quad 20$$

$$c_{\mathrm{eff}} = f_{\mathrm{S}} \cdot c_{\mathrm{S}} + (1 - f_{\mathrm{S}}) \cdot c_{\mathrm{L}} - L\frac{\partial f_{\mathrm{S}}}{\partial T} \quad 21$$

In a similar work, Maurya and Jha (2014) [85] investigated the effect of casting speed in the continuous casting process.

$$\rho\frac{\partial H}{\partial t} + \rho\nabla \cdot (uH) = \nabla\left(k_{eff}\nabla T\right) + Q_\ell \quad 22$$

Where ρ is density, H is enthalpy, ΔH is sensible heat, Q_L is source term. Q_L can be expressed as a single solidification model and given as;

$$Q_L = \rho L\frac{\partial f_s}{\partial t} + \rho L\bar{u}_{pull} \cdot \nabla f_s \quad 23$$

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad 24$$

Naiver-Stokes equation for transient momentum conservation is given by

$$\frac{\partial}{\partial t}(\rho u) + \rho\nabla(uu) = -\nabla P + \nabla\{\mu_{eff}(\nabla \cdot u)\} + \rho + S \qquad \text{(XX)}$$ 25

where,

$\mu_{eff} = \mu_l + \mu_t$

Maurya and Jha (2014) [85] and Hitanen et al. (2017) [86] used the enthalpy-porosity technique for solidification.

$$S = \frac{(1-\beta)^2}{(\beta^3 - \xi)} A_{\text{mush}} \left(\bar{u} - \bar{u}_{\text{pull}}\right)$$ 26

where, liquid fraction is expressed as β, ξ = 0.001, mushy zone constant is given as A_{mush}.

Pilvi et.at., (2017) [65] Used turbulent flow modelling at inlet in which they considered hydraulic diameter at inlet.

$$\lambda_e = \lambda_L L(1 + 6(1 - f_s)^2$$ 27

In 2016, Hibbeler et al. [87] proposed an innovative reduced-order model (ROM) for heat transfer from mold in the continuous casting of steel.

$$0 = \frac{\partial^2 \theta_{\text{mould}}}{\partial x^{*2}} + \left(\frac{d_{\text{mould}}}{w_{\text{mould}}}\right)^2 \frac{\partial^2 \theta_{\text{mould}}}{\partial y^{*2}} + \left(\frac{d_{\text{mould}}}{\ell_{\text{mould}}}\right)^2 \frac{\partial^2 \theta_{\text{mould}}}{\partial z^{*2}}$$ 28

Vnnyscy and Saleem (2017) [88] formulated a mathematical formulation for explicitly calculating the geometrical range of the mushy zone.

$$\rho c_{\text{p}} V_{\text{cast}} \frac{\partial T}{\partial z} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \rho V_{\text{cast}} \Delta H_{\text{f}} \frac{\partial \chi}{\partial z}$$ 29

where

$$k = \chi k_1 + (1 - \chi) k_s$$

$$c_{\text{p}} = \chi c_{\text{pl}} + (1 - \chi) c_{\text{ps}}$$

A decoupled three-dimensional mathematic model of fluid flow and heat transfer in continuous casting billet mould was developed by An et al., (2018) [89].

$$\frac{\partial}{\partial t}(\rho H) + \frac{\partial}{\partial x_j}(\rho \mu_j H) = \frac{\partial}{\partial x_j}\left[\left(\lambda + C_p \frac{\mu_t}{\sigma_t}\right) \frac{\partial H}{\partial x_j}\right]$$ 30

Ole Richter et al. (2017) [90] studied the development of free surface flow for the liquid and/or solid phase change. They considered enthalpy-porosity and volume-of-fluid (VOF) method.

$$\alpha_1 = \qquad 35$$

$$\begin{cases} 0 & = \text{gas} \\ 0 < \alpha_1 < 1 & = \text{cell contains the interface} \\ 1 & = \text{solid or liquid PCM} \end{cases}$$

The molten steel fraction was completely dependent on the thermal condition (T) of liquid metal. TS and TL indicates same respectively. This can be expressed as follows[90];

$$\gamma_{1,l} = \begin{cases} 0 & \text{if} \quad T < T_S \\ \dfrac{T - T_S}{T_L - T_S} & \text{if} \quad Ts \leq T \leq T_L. \\ 1 & \text{if} \quad T > T_L \end{cases} \tag{31}$$

Where one indicates complete liquid state and zero indicates complete solid state. In between values of solid fraction indicates mushy zone.

In the given formulation the density ρ, the heat capacity cp, the heat conduction λ and the viscosity μ can be expressed as follows;

$$\rho = \alpha_1(\gamma_{1,l}\rho_{1,l} + \gamma_{1,s}\rho_{1,s}) + \alpha_2\rho_2 \tag{32}$$

$$c_p = \alpha_1(\gamma_{1,l}c_{p1,l} + \gamma_{1,s}c_{p1,s}) + \alpha_2 c_{p2} \tag{33}$$

$$\lambda = \alpha_1(\gamma_{1,l}\lambda_{1,l} + \gamma_{1,s}\lambda_{1,s}) + \alpha_2\lambda_2 \tag{34}$$

$$\mu = \alpha_1\mu_{1,l} + \alpha_2\mu_2 \tag{35}$$

In above equations, the subscripts [ ]1,l, [ ]1,s and [ ]2 illustatre the property of the bulk liquid, solid and gas phase, respectively. In order to consider natural convection in proposed numerical formulation, the Boussinesq approach was used. Further, the buoyancy modified density ρb can be defined as;

$$\rho_b = \alpha_1(\gamma_{1,l}\rho_{1,l}(1 - \beta(T - T_L)) + \gamma_{1,s}\rho_{1,s}) + \alpha_2\rho_2 \tag{36}$$

3. Conclusions

Recently, Chen et al. (2019) [91] investigated the mold level fluctuations. These fluctuations are caused by transient bulging of the solidifying shell. Consequently, transient bulging phenomenon affects the quality of the steel. They developed a 1D and 2D model for strand simulation. They reported that mold level fluctuations are highly caused by dynamic bulging. Several constitutive models have been adopted for simulating the solidification stresses using the simple elastic-plastic models [92][93]. Many literatures have reported about strand bulging between rolls which have caused transverse cracks, radial streaks and centerline macrosegregation [6], [7], [94]. Risso et al. [95] evaluated the thermal stress and strain in the solidifying shell of the strand by using the analytical method. Researchers added a separate creep model for transient modeling [96]. The integration of these transient constitutive laws and further, mathematical modeling is a challenging task. From all the above discussion it is observed that the temperature and stress-strain distribution in the strand region of the continuous casting process plays an important role in defining the quality of the final solidified product[43], [97]–[99]. A numerical model was presented by Fachinotti et al. (2006) [94]to study the macro-segregation defects in strand caused by thermal stress. They made a hypothesis about the transient effect of alternate rolling and bulging. To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [5]. They compared FPM results with FVM results. It was concluded that heat transfer, surface temperature, and shell thickness can be successfully modeled by FPM method. In 2006, Liu and Zhu [100] developed a three-dimensional finite-element heat-transfer and thermal stress models to study the thermo-mechanical distortion on the slab during operation. They reported that operating parameters i.e., casting affected the strand distortion in copper walls of the mould. Pascon and coworkers (2006) [101] studied the generation of transverse crack during bending and straightening of strands. The numerical model was applied and validated with industrial data. The transverse cracks were found at the upper face of the strand.

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