REVIEW ARTICLE VW Engineering International

DOI: 10.36297/vw.jeit.v3i1.57

VW Engineering International, Volume: 3, Issue: 1, 1-22

# Advancements in numerical modeling of the continuous casting mold

Md Irfanul Haque Siddiqui [1*], Ambrish Maurya [2], Rajneesh Kumar [3]

[1]Department of Mechanical Engineering, King Saud University, Riyadh11421, Saudi Arabia
[2]Department of Mechanical Engineering, National Institute of Technology, Patna, India
[3]Department of Mechanical & Industrial Engineering, Indian Institute of Technology Roorkee, India
*Corresponding Authors Email: msiddiqui2.c@ksu.edu.sa,

Received:
June 23, 2021
Accepted:
Aug. 10, 2021

**Abstract:** This paper presents a critical review of numerical modeling and methods applied in the continuous casting mold. With the recent advancement in metallurgical methods, the continuous casting process now becomes the main method for steel production. To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer, and structural deformation. The important numerical modeling method of the continuous casting process has been discussed in reference in this work. The present work describes molten steel flow, heat transfer, solidification, electromagnetic applications, formation of the shell by solidification and coupling, etc. Further, the distortion of strand by thermo-mechanical forces, bulging, bending, and crack prediction has been discussed briefly. Numerical simulations have led to the path where greater information can be unleashed to understand the metallurgical process of strand solidification.

**Keywords:** steelmaking, mold, mathematical modeling, continuous casting, CFD

## 1. Introduction

More than 96 % of steel products in the world are coming through a continuous casting process [1]. Scientific advancement has been carried out in ironmaking & steelmaking for many decades. Hence, steelmaking is now a well-established technology driven by plant, experimental and computational analysis. With the recent advancement in metallurgical methods, the continuous casting process now becomes the main method for steel production[2], [3]. Nowadays, the old ingot route process has been replaced by the continuous casting process[4] for producing steel in most industries. The reason behind this is the advantages that come with the continuous casting process which includes cost-saving, high productivity, and better quality[5]–[8].

To achieve efficient and effective production, the manufacturers of steel keep on searching for new methods which increase productivity. One such kind of method has become more popular to use optimizing using numerical modeling[9]. The optimized solutions of the temperature distribution that is obtained using a numerical model before the casting process will help avoid the surface defects such as cracks and other unfavorable phenomena[10], [11]. The continuous casting process comprises many complicated phenomena in terms of fluid flow, heat transfer and structural deformation. In a common practice depending on the area of interest for study, researchers simplify the model by focusing on a few phenomenon and ignore the rest [9], [12], [13][14]–[19].

The important part and process of continuous casting have been modeled in-depth and discussed in reference [20]. It describes molten steel flow, formation of the shell by solidification. Further, the distortion of strand by thermo-mechanical forces, bulging, bending and crack prediction has been also given in detail. Numerical simulations have led to the path where greater information can be unleashed to understand the metallurgical process of strand solidification. Apart from manual coding of these problems, various commercial software is available in the market. Till now, many powerful pre-coded solvers are available in the market. The numerical simulation of the thermo-mechanical behavior of the continuous casting process is important in terms of achieving a quality product[21]–[28]. This part of simulation comes with many obstacles such as dealing with the highly non-linear constitutive laws of structure, incorporation of latent heat, involvement of three different states of material: liquid, mushy and solid, temperature-dependent material properties, irregular contact between the mold surface and solidified strand, and coupling the heat transfer and structure model with proper continuum mechanism and boundary condition [29]. Reynold's Averaged Navier– Stokes (RANS) method has been widely adopted for turbulence modeling. It has been reported that the RANS model is highly accurate in predicting steady-state flow patterns [25]. The most adopted technique for simulating the solidification of continuous casting is the enthalpy-porosity approach. This technique is based on a quantity known as a liquid fraction. Many researchers have studied this approach but most of them are limited to 2D modeling. Though modes of heat transfer in the mold are complex phenomena and are studied where modes of heat transfer are conduction, convection & radiation, their effect on the final product and the possibility of controlling the detrimental effect has to be studied.

The research work done in the last three decades has made continuous casting an advanced and sophisticated technology[20], [30]–[32]. Physical water models can simulate the molten steel flow in the mold region of the continuous casting process considering the viscosity of water equivalent to steel [33] [34]. To overcome the inaccuracies of the water model, researchers have developed numerical models based on finite-volume methods to get the solution of Navier-Stokes's equations. The characteristic of molten steel flow, heat transfer, and solidification in the mold have been studied in previous research [35]–[37]. Several research works have been done on molten steel flow, heat transfer and solidification in mold[38]–[40]. The characteristic of molten steel flow, heat transfer and solidification in the mold have been studied in previous research [35]–[37]. These studies have been established and validated with industrial trials[9], [30], [41]. Moreover, experimental and numerical investigations have been dedicated to solidification and thermo-mechanical deformation in strands. Research has been done on temperature distribution and thermo-mechanical deformation of the strand in the continuous casting process[42]–[44]. From all previous studies, it is well established that numerical models efficiently and accurately predict the fluid flow and mechanical behavior of mold and strand, respectively [33] [34]. Recently, much effort is given to studies on numerical modeling of thermo-mechanical deformation in continuous casting strands.

In this present work, we have reviewed the literature to provide current information on the mathematical modeling of steelmaking tundish. This review work embraces various aspects of mathematical modeling of continuous casting molds such as melt flow, turbulence modeling, heat transfer, solidification, electromagnetic applications, thermo-mechanical deformation and coupled models, etc.

## 2. Melt flow modeling

The molten steel flow in continuous casting mold is usually assumed to have some characteristics. These flow characteristics are classified based on some assumptions such as compressible and incompressible. The molten steel flow is governed by the continuity equation and momentum equation, supplemented by heat transfer boundary conditions[45]–[47]. The governing equations related to mass flow and momentum transfer are as follows [48];

$$\nabla \cdot u = 0 \qquad 1$$

$$\rho\left(\frac{\partial u}{\partial t} + (u \cdot \nabla)u\right) = f - \nabla p + \mu\nabla^2 u \qquad 2$$

In 2005, Zhao et al. [49] studied the transient molten steel flow and superheat transport in a continuous casting mold. In the proposed model, momentum equations and the energy equation were coupled. The coupling was done by buoyancy term in the z momentum equation, using the Boussinesq approximation. The equation for momentum calculation was used as follows.

$$\rho_0\left(\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial\left(\bar{u}_i \bar{u}_j\right)}{\partial x_j}\right) = -\frac{\partial \bar{p}}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \bar{u}_i}{\partial x_j}\right) - +\delta_{i3}\rho_0\beta(\bar{T} - T_0)g \qquad 3$$

where $\bar{u}_i$ represents "grid-filtered" velocities in the $x, y$, and $z$ directions (represented by $i, j = 1,2,3$), $\bar{p}$ is the pressure, $\bar{T}$ is the temperature, $\mu$ is the dynamic viscosity, $\rho_0$ is the density at the reference temperature $(T_0)$, $\beta$ is thermal-expansion coefficient, $g$ is the gravitational acceleration, $k$ is the thermal conductivity, $C_p$ is the heat capacity, and $\delta_{i3}$ is the Kronecker.

Thermal expansion coefficient expressed by β, thermal conductivity by k, Kronrcker term by $\delta_{ij}$, and heat capacity by $C_p$. The used finite-volume method was used to solve equations along with central differencing with second-order accuracy. They did time integration of the equations by using a semi-implicit, fractional-step method. Further, diffusion terms were used implicitly by the Crank–Nicolson method. Further momentum equation was modified by using a sub-grid momentum flux term $Q_{ij}$.

where,

$$Q_{ij} = \rho_0\left(\bar{u}_i\bar{u}_j - \bar{u}_i\bar{u}_j\right) \quad 4$$

### 3. Turbulence models

Most of the previous work on continuous casting mold has been modeled using the RANS equation [49]. Therefore, turbulent viscosity was predicted for the Large Eddy Simulation (LES) model from the following equation.

$$\mu_T = C_v\rho_0 K_G^{1/2}\Delta \quad 5$$

where the constant $C_v$ is $0.05$, and $\Delta$ is the grid-length scale, given by $\Delta = \left(\Delta_x\Delta_y\Delta_z\right)^{1/3}$($\Delta_x, \Delta_y$, and $\Delta_z$ are grid sizes in the $x, y$, and $z$ directions, respectively).

To understand the complex flow profile in mold, Li and Tsukihashi [50] have developed a numerical model to investigate the vortexing flow in SEN of continuous casting of steel. Further, it was reported that the vortex flow came into existence from 3d molten steel flow in the mold. To describe the behavior of vortices in the flow field, definitions of the vorticity $\omega_z$ was expressed as follows,

$$\omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial \theta}\right) \quad 6$$

The superheated steels remain in the liquid state. At this state, liquid steel is considered Newtonian. In 2011, Sowa and Bokota [51] have assumed viscous incompressible and laminar flow to describes flow patterns in mold. They proposed the following equation for mass and momentum calculation.

$$\nabla . v = 0 \quad 7$$

$$\rho\frac{d\mathbf{v}}{dt} = \rho\mathbf{g} - \nabla p + \mu\nabla^2\mathbf{v} \quad 8$$

Here, in the above equation flow was assumed incompressible and laminar. In a similar work in 2013, Zare et al.[52] investigated the molten steel flow filed in the mold under various conditions of submerged entry nozzle. In their work, the following momentum equation was solved;

$$\frac{\partial(\rho v_i v_j)}{\partial X_i} = -\frac{\partial p}{\partial X_i} + \frac{\partial}{\partial X_i}\left[\mu_{eff}\left(\frac{\partial v_i}{\partial X_i} + \frac{\partial v_i}{\partial X_j}\right)\right] + \rho g_j + F_j \quad 9$$

In the above equation, Zare et al. (2013)[52] used the effective viscosity term in the momentum equation. The terms k and ε for turbulent viscosity were predicted from two equations of the standard k-ε model. It was expressed as follows.

$$\mu_{eff} = \mu + \mu_t \quad 10$$

$\mu_t$ can be calculated using $k - \varepsilon$ parameters:

$$\mu_t = C_\mu\rho\frac{k^2}{\varepsilon} \quad 11$$

In 2015, Ren et al. [46] used a mushy zone parameter ($A_{mush}$) in the momentum equation to incorporate the mushy zone in the calculation. The modified equation for momentum was given as follows [46];

$$\frac{\partial}{\partial t}(\rho u) + \nabla\cdot(\rho u u) = -\nabla p + \nabla\cdot[(\mu_\ell + \mu_t)\nabla u] + \rho g + \rho g\beta(T - T_0) + \frac{\left(1 - f_{\mathrm{liq}}\right)^2}{f_{\mathrm{liq}}^3 + 0.001}A_{\mathrm{mush}}\left(u - u_s\right) + F_{\mathrm{ave}} \quad 12$$

In the above equation, $f_{liq}$ the liquid fraction. The lever rule of solidification was utilized to calculate the mushy zone as follows.

$$f_{liq} = 1 - \frac{1}{1 - k_0}\frac{T - T_{liq}}{T - T_{melt}} \quad 13$$

In 2019, Chen et al. [53] investigated a coupled three-dimensional model of the mold and the numerical model was based on Large Eddy Simulation (LES). One such popular model that works on the above-mentioned method is the k-ε model Further, more details on the mathematical modeling of multi-phase fluid flow can be read elsewhere [22], [54], [55].

Numerical solution of these equations is virtually not possible by Direct Numerical Simulation (DNS) due to its requisite of extremely high computational memory, time, and power. For this reason, turbulence modeling is carried out to imitate the effects of turbulent fluid flow patterns. Comparative studies have been carried out by many researchers to investigate the influence of various turbulence models on the estimation of results [56]–[61]. Siddiqui et al., [62] compared different turbulence models and predicted results revealed that the RNG k-ε model has a good approximation of F-curves as well as the interface between the two phases. Predictions made by all models except the SST k-ω model have shown a satisfactory approximation with the experimental values. In 2013, Liu et al. investigated the Large Eddy Simulation model to investigate the slab caster mold flow. They reported that simulation results were satisfactorily matched with experimental results. The equations of motion for $k$ phase in an Euler-Euler simulation are generally given as follows:

$$\frac{\partial(\alpha_k\rho_k)}{\partial t} + \nabla\cdot(\alpha_k\rho_k u_k) = 0 \qquad 14$$

$$\frac{\partial(\alpha_k\rho_k u_k)}{\partial t} + \nabla\cdot(\alpha_k\rho_k u_k u_k) = -\nabla\cdot(\alpha_k\tau_k) - \alpha_k\nabla P + \alpha_k\rho_k g' + M_{l,k}$$

The stress terminology of k phase can be written as:

$$\tau_k = -\mu_{eff,k}\left(\nabla u_k + (\nabla u_k)^T - \frac{2}{3}\mathrm{I}(\nabla\cdot u_k)\right) \qquad 15$$

In the above equation, effective viscosity is defined as $\mu_{eff}$. This term can be explained in form of different viscosity, namely molecular and turbulent viscosity along with some arbitrary terms due to turbulence formation by bubbles.

$$\mu_{eff} = \mu_{L,I} + \mu_{T,1} + \mu_{Bl,1} \qquad 16$$

Empirically the calculation of effective viscosity of gas was calculated from effective liquid velocity.

$$\mu_{ef,g} = \frac{\rho_g}{\rho_l}\mu_{eff,l} \qquad 17$$

The model proposed by Sato & Sekiguchi $^{22}$ has been used to take account of the turbulence induced by the movement of the bubbles. The expression is:

$$\mu_{BI,l} = \rho_i C_{\mu,BI}\alpha_g d_g\left|u_g - u_i\right| \qquad 18$$

with a model constant $C_{\mu,Bu}$, which is equal to $0.6$. Turbulent flows are characterized by eddies with a wide range of length and time scales. The largest eddies are typically comparable in size to the characteristic length of the mean flow. The smallest scales are responsible for the dissipation of turbulence kinetic energy. The velocities in Eqs. (1) and (2) are defined as follows:

$$u_k = \tilde{u}_k - u'_k \qquad 19$$

A numerical model has been developed to analyze the transient three-dimensional and three-phase flow in a bottom stirring ladle with a centered porous plug, which takes into account the steel, gas, and slag phases; it enables us to predict the fluid flow and heat transfer in the very important steel/slag region. They applied k-ε turbulence model[63];

$$\frac{\partial(\rho k)}{\partial t} + \rho u_j\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu_{\mathrm{eff}}}{\sigma_k}\cdot\frac{\partial k}{\partial x_j}\right) + G_k + G_b - \rho\varepsilon \qquad 20$$

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \rho u_j\frac{\partial\varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu_{\mathrm{eff}}}{\sigma_\varepsilon}\cdot\frac{\partial\varepsilon}{\partial x_j}\right) + \frac{(c_1G_k\varepsilon + c_2G_b - c_3\rho\varepsilon^2)}{k} \qquad 21$$

In the above relationship, $G_k$ is the turbulent kinetic energy generated by mean flow velocity gradients. This can be written as follows;

$$G_k = \mu_t\frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad 22$$

$$G_k = \mu_t\frac{\partial u_j}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad 23$$

Further, $G_b$ shows the turbulent kinetic energy generated by buoyancy and it can be expressed as;

$$G_b = -g\frac{\mu_t}{\rho\mathrm{Pr}_t}\frac{\partial\rho}{\partial x_i} \qquad 24$$

The effective viscosity can be written as the addition of laminar and turbulent viscosities, as follows.

$$\mu_{\mathrm{eff}} = \mu + \mu_t = \mu + \rho c_\mu\frac{k^2}{\varepsilon} \qquad 25$$

The values for the constants in this $k - \varepsilon$ model $c_1, c_2, c_3, c_u, \sigma_k$, and $\sigma_\varepsilon$ are $1.43, 1.92, 0.09, 1.00$, and 1.30, respectively [15].

In 2014, Li et al. [64] developed a mathematical model to study the vortex formation in ladles. It is formed during liquid steel teeming from the ladle. They studied vortex formation during ladle teeming using new technology. The results obtained help to verify the validity of the numerical computations.[64]

Turbulent kinetic energy equation $(k)$:

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left(\alpha_k \mu_{eff}\frac{\partial k}{\partial x_j}\right)+ G_k+G_b-\rho\varepsilon-Y_M+S_k \tag{26}$$

Turbulent dissipation rate equation ($\varepsilon$)

$$\frac{\partial}{\partial t}(\rho \varepsilon)+\frac{\partial}{\partial x_i}(\rho \varepsilon u_i)=\frac{\partial}{\partial x_j}\left(\alpha_\varepsilon \mu_{eff}\frac{\partial \varepsilon}{\partial x_j}\right)+ C_{1\varepsilon}\frac{\varepsilon}{k}(G_k+C_{3\varepsilon}G_b)-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}-R_\varepsilon+S_\varepsilon \tag{27}$$

In the past, various viscosity models have been used by the researchers to take care of the turbulence flow in the continuous casting process such as effective viscosity models[65] which are useful for cylindrical mold and lined nozzle, one equation turbulence models which are based on turbulent energy spread over a given length-scale [66], two-equation turbulence models such as K-ε model, K-ω model,[67][68], Large Eddy Simulation (LES) [69] model and Direct Numerical Simulation (DNS) [70]. While selecting the turbulence model for the simulation, one needs to take care of the computational requirement as well and the easiness of modeling. Few authors have also used a large eddy simulation (LES) model for transient analysis[69]. LES turbulent model is effective in producing transient flow [71]–[73]. In 2019, Chen et al. [53] investigated a coupled three-dimensional model of the mold and the numerical model was based on Large Eddy Simulation (LES) [74] [16], [59], [74]–[80].

## 4. Heat Transfer and Solidification

The fundamental requirement of the continuous casting process is to solidify the strand to achieve plant set quality standards [20], [81], [82]. Specifically, the solidification process is heat removal from high-temperature liquid metal and semi-liquid metal to produce solid strands. Alizadeh et al. (2006) [83] developed a finite volume-based numerical model to investigate heat transfer and solidification mechanisms in real plant slab casters. The most adopted technique for simulating the solidification of continuous casting is enthalpy-porosity[84]–[86]. This technique is based on a quantity known as a liquid fraction. Several researchers used industrial plant data to accurately model molten steel flow and heat transfer phenomenon [32], [87], [88]. They studied mass and energy balance, the temperature difference of cooling-water, mold temperature, steel shell thickness, etc.[87], [89]. Alizadeh et al. (2006) [83] developed a finite volume-based numerical model to investigate heat transfer and solidification mechanisms in real plant slab casters. The numerical model solved the two-dimensional transient heat transfer equation. They applied proper boundary conditions for mold, water jet spray cooling region and ambient cooling for strand. The numerical model results were compared with plant data and it predicted temperature distribution and solidified shell thickness. Generalized heat transfer equation (3-dimension) can be written in the most suitable format from the above equations in the following manner[48];

$$\rho C\left(\frac{\partial T}{\partial t}+V_x\frac{\partial T}{\partial t}+V_y\frac{\partial T}{\partial t}+V_z\frac{\partial T}{\partial t}\right)=\ddot{q}+ \frac{\partial}{\partial x}\left(K_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right) \tag{28}$$

In 2005, Louhenkilpi et al. [90] proposed a three-dimensional transient formulation for temperature distribution over the mold wall. They developed a three-dimensional DYN3D finite-difference model for heat transfer calculation in continuous casting. They considered the gap heat transfer coefficient as a function of the strand surface temperature was used to couple mold and strand model. The heat transfer equation was expressed as follows;

$$\rho\frac{\partial H}{\partial t}+v\frac{\partial H}{\partial z}=\frac{\partial}{\partial x}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_{\mathrm{eff}}\frac{\partial T}{\partial z}\right) \tag{29}$$

In a similar work, Zhao et al. (2005) [49] modeled energy equation along with the Navier-Stokes equation. The energy equation was modified with the derivative of the subgrid heat flux term. However, it was found that using the static subgrid-scale model into the 3d finite-volume code had minimal impact. The QTi represents the SGS heat fluxes in the following equation for heat transfer;

$$\frac{\partial \bar{T}}{\partial t}+\frac{\partial(\bar{u}_i\bar{T})}{\partial x_i}=\frac{k}{\rho_0 C_P}\frac{\partial}{\partial x_i}\left(\frac{\partial \bar{T}}{\partial x_i}\right)+\frac{\partial Q_{Ti}}{\partial x_i} \tag{30}$$

$$Q_{Ti}=\bar{T}\bar{u}_i-\overline{Tu_i} \tag{31}$$

where, T is temperature and $u_i$ represents velocities in x, y, and z-direction. The heat flux ($Q_{TI}$) was considered as an extra diffusion term. It was calculated from the eddy-viscosity value $\mu_T$ from the momentum equation and the turbulent Prandtl number ($Pr_T = 0.9$):

$$\frac{\partial Q_{Ti}}{\partial x_i}=\frac{\mu_T}{\mathrm{Pr}_T}\frac{\partial}{\partial x_i}\frac{\partial \bar{T}}{\partial x_i} \tag{32}$$

In 2011, Sowa and Bokota [51] proposed a heat flow model based on the Fourier-Kirchhoff system of equations with the convection term along with the volumetric efficiency of the internal heat source (Q). In the mathematical model, they considered temperature-dependent thermophysical parameters.

Further, they also considered the solid/liquid phase volume fractions in the mushy zone. They called this formulation complex because the model incorporated the solid, liquid and mushy phases in the calculation of mass and momentum transfer. The advancing solidification boundary was considered as mushy, i.e. the molten steel is being solid within the range of solidus and liquidus temperature. The heat flow equation described the heat flow in liquid and solid-state of steel;

$$\rho c\left(\frac{\partial T(\mathbf{x},t)}{\partial t}+\nabla T\cdot\mathbf{v}\right)=\nabla\cdot(\lambda\nabla T)+\dot{Q} \qquad 33$$

Where internal heat source volumetric efficiency is defined as Q [W/m3],

Hence, the heat generation term is not valid in the case of thermal conductivity calculation in the solid phase. Moreover, temperature-dependent properties have been not considered in the above equation. Therefore, Sowa and Bakota et al. [51] modified the above equation which includes effective specific heat (Ceff) term which is a function of the temperature of the material.

$$\nabla.(\lambda\Delta T)-C_{ef}\frac{\partial T}{\partial t}-C_{ef}\nabla T.V=0 \qquad 34$$

$$C_{ef}(T)=\rho_{LS}c_{LS}+\rho_S L/(T_L-T_S) \qquad 35$$

Where $C_{ef}$ the effective heat capacity of the mushy zone in mold and secondary cooling zone.

$L$ – the latent heat of solidification,

$C_{LS}$ – the specific heat of the mushy zone,

$\rho_S$, $\rho_L$, $\rho_{LS}$ – the density of solid phase, liquid phase, and mushy zone, respectively

In 2011, Hadata et al., [39] proposed a steady Fourier-Kirchhoff model for heat flow with some assumptions. They calculated mold temperature from the 3d transient simulation model. In this work, the volumetric heat generation term ($q_v$) was expressed as follows;

$$q_v=Q_S\frac{dV_S}{d\tau} \qquad 36$$

where: Qs heat of solidification, for steel Qs = 1.9 109 J/m3; Vs – solid phase volume fraction, time is expressed as τ.

In a study in 1993 S.E.Chidiac et.at.,[64]used enthalpy approach for heat transfer in multi-dimensional problem with following equation.

$$\rho\frac{\partial H}{\partial t}=\nabla\cdot(K\nabla T)+Q \qquad 37$$

where $\rho$ indicates density, $H$ indicates enthalpy, $K$ indicates Thermal conductivity, $Q$ indicates heat generation rate for unit volume, $T$ indicates temperature and $t$ time. Enthalpy is nothing but the summation of sensible & latent heat and can be expressed as:

$$H=\int_{T_r}^{T}c\,dT+f(T).L \qquad 38$$

where c*,* *f(T)* and *L* are specific heat liquid fraction and latent heat. For phase change study two methods are clubbed together with the above-stated formulation for accuracy and efficiency. Dirichlet & Cauchy boundary conditions are used to solve above equations. The study carried in 2003, B. wiwanapataphee et. al., [63] for simulating phase change cause of heat transfer single domain enthalpy method is adopted. Where enthalpy is the summation of latent heat (H) & sensible heat (h).

$$H=h+\Delta H \qquad 39$$

Latent heat h can be given by

$$H=f(T)L, \qquad 40$$

Where L denoted Latent Heat of Steel *L* and *f(T)* indicates localized liquid fraction where value one represents complete Liquids state and zero represents the complete solid-state. The liquid fraction is nonlinear for simplification of the model it is assumed linear.

$$f(T)=\begin{cases}0, & T\leqslant T_s\\ \frac{T-T_s}{T_L-T_s}, & T_s<T<T_L,\\ 1, & T\geqslant T_L,\ (XX)\end{cases} \qquad 41$$

wherein $T_L$ indicates melting temperature and $T_S$ Solidification temperature

For region where phase change occurs conservation of energy principle. Combining this equation with enthalpy gives,

$$\rho c\left(\frac{\partial T}{\partial t}+u_jT_,\right)=\left(k_0T_j\right)_j-S_T \qquad 42$$

where $u_j$ indicates velocity in x direction, ρ indicates density, $k_0$ thermal conductivity of steel, $S_T$ represents rate change in volumetric latent heat. If $S_T$ is in between zero and one, indicates region where change in phase occurs. Using the above approach in modeling eliminates consideration of solidified and liquid regions without considering phase change boundary. Thus making equation generalized and can be used to numerically model all three domains which are solid, liquid and mushy region. With assumptions of a linear liquid fraction, a single model is proposed to model all three phases of solidification.

Ivanova (2013) [91] formulated extensive mathematical modeling on predicting phase-dependent boundary conditions. The boundary position between liquid and solid phase was given by the temperature equality condition and the Stefan condition for the two-dimensional case. They proposed a method based on the finite differences using explicit schemes and on the iteration method of solving non-linear system equations. The heat transfer from the ingot was expressed as follows;

$$\frac{\partial T}{\partial r} + v(t) \cdot \frac{\partial T}{\partial z} = \frac{1}{c(T)\rho(T)} \times \\ \times \left\{ \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial \partial_z}\left[\lambda(T)\frac{\partial T}{\partial_z}\right] \right\} \quad 43$$

where $t$ is a time, $x, z$ are space variables (Fig 1 ), $v(t)$ is the casting speed. $T = T(\tau, x, z)$ is the temperature of ingot metal. $c(T)$ is its specific heat, $\rho(T)$ is its density, and $\lambda(T)$ is its thermal conductivity.

The position of the unknown phase boundary is specified by the equality condition of the temperatures and the Stefan condition for the two-dimensional case:

$$T = \\ T(\tau, x, z)|_{x=\xi\_(\tau,z)=} T(\tau, x, z)|_{x=\xi_+(\tau,z)=T_{cr}} \quad 44$$

$$\lambda(T)\ \frac{\partial T}{\partial \bar{n}}|\xi_+ - \ \lambda(T)\frac{\partial T}{\partial \bar{n}}|\xi_- = \\ \mu\rho\ (T_{kp})\ \left(\frac{d\xi}{d\tau} + v(\tau)\frac{d\xi}{dz}\right) \quad 45$$

where $\xi$ is the phase boundary $x = \xi(\tau, z), \bar{n}$ is a normal to the phase boundary, $\left.\frac{\partial T}{\partial \pi}\right|_{\xi+/-}$ is the left-right limit of the temperature derivative in the normal direction. $\mu$ is the latent the heat of crystallization. $T_{cr}$ is the crystallization temperature (the average temperature from the liquidus-solidus interval).

To solve phase-change boundary conditions, they replaced derivatives in the differential equations using the corresponding difference relations, i.e. linear combinations of the values of the grid function in several nodes. Additionally, they considered the set of nodes to determine the phase-change boundary position. In 2014, Zhang et al [92] investigated a steady-state two-dimensional numerical model based on the assumption of heat transfer in casting direction is negligible or small and melt surface has a uniform temperature of pouring. The stated model significantly differed from previous models in terms of considering material properties. They applied a temperature-dependent steel density and conductivity as following. However, the density, conductivity and equivalent specific heat are dependent on a range of temperature i.e., phase-dependent.

$$\rho = (1 - f_\mathrm{S})\rho_\mathrm{L} + f_\mathrm{S}\big(f_\delta\rho_\delta + f_\gamma\rho_\gamma\big) \quad 46$$

$$\lambda = (1 - f_\mathrm{S})\lambda_\mathrm{L} + f_\mathrm{S}\big(f_\delta\lambda_\delta + f_\gamma\lambda_\gamma\big) \quad 47$$

$$c_\mathrm{eff} = f_\mathrm{S} \cdot c_\mathrm{S} + (1 - f_\mathrm{S}) \cdot c_\mathrm{L} - L\frac{\partial f_\mathrm{S}}{\partial T} \quad 48$$

In a similar work, Maurya and Jha (2014) [93] investigated the effect of casting speed in the continuous casting process. Their study was based on the investigation of heat transfer and solidification behavior of steel within mold and Secondary Cooling Zone (SCZ). A source term $Q_L$ was introduced in the energy conservation equation which has two parts namely explicit latent heat term and convective term. They used an energy conservation approach for studying solidification.

$$\rho\frac{\partial H}{\partial t} + \rho\nabla \cdot (uH) = \nabla\big(k_{eff}\nabla T\big) + Q_\ell \quad 49$$

Where ρ is density, H is enthalpy, ΔH is sensible heat, $Q_L$ is source term. $Q_L$ can be expressed as a single solidification model and given as;

$$Q_L = \rho L\frac{\partial f_s}{\partial t} + \rho L\bar{u}_{pull} \cdot \nabla f_s \quad 50$$

Latent heat is released from the mushy zone. One indicates the solid-state and will go down with casting speed. The total sum of solid fraction & liquid fraction is always equal to one. Further continuity equation can be given as

$$\frac{\partial}{\partial x_i}(\rho u_i) = 0 \quad 51$$

Naiver-Stokes equation for transient momentum conservation is given by

$$\frac{\partial}{\partial t}(\rho u) + \rho\nabla(uu) = -\nabla P + \nabla\big\{\mu_{eff}(\nabla \cdot \\ u)\big\} + \rho + S \qquad \text{(XX)} \quad 52$$

where,

$\mu_{eff} = \mu_l + \mu_t$

$\mu_{eff}$ is corrected viscosity because of turbulence, $\mu_l$ is dynamic viscosity, $\mu_t$ is turbulent viscosity. In porosity-enthalpy formulation mushy zone is made porous and the same porosity is assigned to each cell which is the same as the liquid fraction for that cell. Hence porosity equal to one indicates a complete solid-state and equals to zero indicates a complete liquid state.

They added one more term S in the Naiver-Stokes equation called the momentum shrink term. This helps to add movement of solidified material to the bottom of casting with a velocity equal to molten metal and treated constantly. Maurya and Jha (2014) [93] and Hitanen et al. (2017) [94] used the enthalpy-porosity technique for solidification. In their model, the porous medium was considered as mushy zone and the liquid fraction was represented by porosity fraction. In solid-phase porosity equals to zero, which diminishes the velocity in that zone. Therefore, momentum sink term "S" was used in Navier- Stokes equation as follows;

$$S = \frac{(1-\beta)^2}{(\beta^3-\xi)} A_{\text{mush}} \left(\bar{u} - \bar{u}_{\text{pull}}\right) \qquad 53$$

where, liquid fraction is expressed as β, $\xi = 0.001$, mushy zone constant is given as $A_{\text{mush}}$.

Pilvi et.at., (2017) [65] Used turbulent flow modelling at inlet in which they considered hydraulic diameter at inlet. With RNG κ-ε approximate turbulence intensity calculated is 4.3%. They adjusted mass flow rate to match the casting speed. Further found that flow changes from turbulent to laminar as it changes very fast, hence use of RNG κ-ε model justified. T.Telejko et.al., (2009) [66] they studied three heat transfer model for CC process. First model considered is convection in which the solidified metal goes down with velocity of casting velocity. In second model convection is considered by arbitrarily increasing thermal conductivity of steel in such way that following relation satisfies.

$$\lambda_e = \lambda_L L(1 + 6(1 - f_s)^2 \qquad 54$$

where $\lambda_e$ equivalent thermal conductivity of non-solidified steel, $\lambda_l$ thermal conductivity of liquid steel, $f_s$ fraction of solidified steel. In third model they keep thermal conductivity same like first model. Velocity profiles are calculated using Navier-Stokes equation. First & third model gives approximate same cooling effect. Beginning of steel shell meniscus formation and its completion is nearly same. In second model delayed formation of steel shell meniscus and completion observed. Effective thermal coefficient that is model second is not accurate.

In 2016, Hibbeler et al. [95] proposed an innovative reduced-order model (ROM) for heat transfer from mold in the continuous casting of steel. The numerical formulation was attached with the solidification model. Further, it also considered mold-metal interfacial phenomena. It was able to accurately and efficiently produce heat transfer and solidification in the continuous casting process. The scaled heat transfer equation was given as;

$$0 = \frac{\partial^2 \theta_{\text{mould}}}{\partial x^{*}2} + \left(\frac{d_{\text{mould}}}{w_{\text{mould}}}\right)^2 \frac{\partial^2 \theta_{\text{mould}}}{\partial y^{*2}} + \left(\frac{d_{\text{mould}}}{\ell_{\text{mould}}}\right)^2 \frac{\partial^2 \theta_{\text{mould}}}{\partial z^{*2}} \qquad 55$$

The proposed ROM was a novel numerical formulation that has minimized the complication of mathematical models. At the same time, it retains the most important behaviors of the numerical model. Further, that model was able to maintain the relationship between inputs and outputs. Vnnyscy and Saleem (2017) [96] formulated a mathematical formulation for explicitly calculating the geometrical range of the mushy zone. It is considered a sharp-interface formulation to predict a dual moving boundary problem to locate the solidus and liquidus isotherms. Heat transfer in the solid, liquid and mush regions is expressed as follows;

$$\rho c_{\text{p}} V_{\text{cast}} \frac{\partial T}{\partial z} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \rho V_{\text{cast}} \Delta H_{\text{f}} \frac{\partial \chi}{\partial z} \qquad 56$$

where

$$k = \chi k_1 + (1 - \chi) k_s$$

$$c_{\text{p}} = \chi c_{\text{pl}} + (1 - \chi) c_{\text{ps}}$$

A decoupled three-dimensional mathematic model of fluid flow and heat transfer in continuous casting billet mould was developed by An et al., (2018) [97]. They modified the energy balance equation for continuous casting mould. They introduced ratio of turbulent viscosity ($\mu_t$) and turbulent Prandtl number ($\sigma_t$) along with enthalpy term $H$ in equation. However, the viscosity, the specific heat, and the thermal conductivity were assumed to be constant over temperature. In this model, value of σt was assumed to unity, however, Zhao et al., used σt=0.9 in their model.

$$\frac{\partial}{\partial t}(\rho H)+\frac{\partial}{\partial x_j}(\rho\mu_j H)=\frac{\partial}{\partial x_j}\left[\left(\lambda+C_p\frac{\mu_t}{\sigma_t}\right)\frac{\partial H}{\partial x_j}\right] \tag{57}$$

Ole Richter et al. (2017) [98] studied the development of free surface flow for the liquid and/or solid phase change. They considered enthalpy-porosity and volume-of-fluid (VOF) method. These methods were coupled in numerical formulation. In proposed model, they introduced free surfaceof liquid metal along with phase change material (PCM). The volume of fluid (VOF) model was introduced in finite volume model. They studied three phases of material namely, airin gaesousss phase and steel in solid and liquid phase. [98].

$$\alpha_1 = \begin{cases} 0 & = \text{gas} \\ 0<\alpha_1<1 & = \text{cell contains the interface} \\ 1 & = \text{solid or liquid PCM} \end{cases} \tag{58}$$

The molten steel fraction was completely dependent on the thermal condition (T) of liquid metal. TS and TL indicates same respectively. This can be expressed as follows[98];

$$\gamma_{1,l} = \begin{cases} 0 & \text{if} \quad T<T_S \\ \dfrac{T-T_S}{T_L-T_S} & \text{if} \quad Ts \le T \le T_L. \\ 1 & \text{if} \quad T>T_L \end{cases} \tag{59}$$

Where one indicates complete liquid state and zero indicates complete solid state. In between values of solid fraction indicates mushy zone.

In the given formulation the density ρ, the heat capacity cp, the heat conduction λ and the viscosity μ can be expressed as follows;

$$\rho = \alpha_1(\gamma_{1,l}\rho_{1,l}+\gamma_{1,s}\rho_{1,s})+\alpha_2\rho_2 \tag{60}$$

$$c_p = \alpha_1(\gamma_{1,l}c_{p1,l}+\gamma_{1,s}c_{p1,s})+\alpha_2 c_{p2} \tag{61}$$

$$\lambda = \alpha_1(\gamma_{1,l}\lambda_{1,l}+\gamma_{1,s}\lambda_{1,s})+\alpha_2\lambda_2 \tag{62}$$

$$\mu = \alpha_1\mu_{1,l}+\alpha_2\mu_2 \tag{63}$$

In above equations, the subscripts [ ]1,l, [ ]1,s and [ ]2 illustatre the property of the bulk liquid, solid and gas phase, respectively. In order to consider natural convection in proposed numerical formulation, the Boussinesq approach was used. Further, the buoyancy modified density ρb can be defined as;

$$\rho_b = \alpha_1(\gamma_{1,l}\rho_{1,l}(1-\beta(T-T_L))+\gamma_{1,s}\rho_{1,s})+\alpha_2\rho_2 \tag{64}$$

## *5. Thermo-mechanical Deformation*

The behavior of metal especially steel at high temperature becomes sensitive to strain rate and temperature. Therefore, process design of hot metal working of steel is significantly affected by non-linear behavior of steel. Structural distortion arises in mold and strand due to thermal distribution, which causes thermal stress, cracks and ultimately affects quality strand [37], [99]. Many research has been done on mould thermal distortion in mould and strand [9], [10], [100]–[102]. In 2006, To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [3]. It has been also reported heat transfer rate is affected by mold distortion[100], [103]. Many research has been done on mould thermal distortion in mould and strand [100], [101]. Generally, the heat transfer equation is solved with interfacial heat flux data and it is quantified from plant data. Subsequently, equations related to thermo-mechanical distortion in mold and strand is calculated.

In some of the early work, Samarasekera et al. (1983) [100] summarizes the deformation in strand due to elastic-plastic behavior. It was characterized in two fundamental relationships. In first yield criteria, it was determined that whether the applied stress could cause yield from an elastic to a plastic condition of solid strand. Secondly, (from the flow law), it correlates advancing stresses and strains in the yielded condition. Thus, elasto-plastic deformation are composed of two components during material undergoing in strain;

$$d\varepsilon_{ij} = d\varepsilon_{ij}{}^{e} + d\varepsilon_{ij}{}^{p} \tag{65}$$

where $d\varepsilon_{ij}{}^{e}$ and $d\varepsilon_{ij}{}^{P}$ are the incremental elastic and plastic components of the total strain vector $d\varepsilon_{ij}$

In this work they proposed incremental stresses and strains during plastic flow;

$$d\varepsilon_{ij}{}^{P} = d\lambda\frac{\partial Y}{\partial\sigma'_{ij}} \tag{66}$$

where dλ is a scalar multiplying factor, dY is derivative of yield stress and $\sigma_{ij}$ is the deviatoric stress vector.

In 2000, Lee et al. [104] proposed a modified model of thermo-mechanical deformation in strand. They developed a mathematical model for the coupled analysis. The coupled analysis consisted of various mathematical models. The coupled model considered molten steel flow characteristics in mould. Further, it coupled the and heat transfer, thermo-mechanical deformation behavior of a solidifying strand in the continuous casting process. Moreover, Von-mises yield function and associated flow were assumed for increment of stress. The stress in thermo-elasto-plastic material can related as;

$$\sigma_{ij} = C_{ijkl}\left(\varepsilon_{kl} - \varepsilon_{kl}^{P} - \varepsilon_{kl}^{Th}\right) \qquad 67$$

where $C_{ijkl}, \varepsilon_{kl}, \varepsilon_{kl}^{P}$, and $\varepsilon_{kl}^{Th}$ are the elastic constitutive matrix, total infinitesimal strain, plastic strain, and thermal strain, respectively.

In a similar work, Ha et al., (2000) [81] carried a mathematical modeling for heat transfer study in secondary cooling zone of continuous casting strand. It was reported that creep was dominant factor in bulging defect. The elastic-plastic creep model for the strand is given by:

$$\dot{\varepsilon} = \alpha\sigma^{m} \qquad 68$$

where $\sigma(\mathrm{kg/cm^2})$ and $\dot{\varepsilon}(\mathrm{I/s})$ denote the equivalent stress and the creep strain rate, respectively, and $m$ is a constant of $3.15$. Also

$$\alpha = 0.0806\exp\left\{-\frac{28392}{T+273}\right\} \qquad 69$$

In 2004, Bellet et al. [10] introduced a global non-steady state (GNS) method for liquid-solid constitutive model which considered mushy zone during solidification. They reported the following relationship for total strain calculation in liquid and mushy zone;

$$\dot{E} = \dot{\varepsilon}^{\mathrm{vp}} + \dot{\varepsilon}^{\mathrm{th}} \qquad 70$$

where Evp is a strain in visco-plastic condition and Eth strain due to thermal expansion. In addition to this, a thermo-elastic–viscoplastic model was used to represent the behavior in the solid state. It was described by the following equations[10];

$$\dot{\varepsilon} = \dot{\varepsilon}^{\mathrm{el}} + \dot{\varepsilon}^{\mathrm{vp}} + \dot{\varepsilon}^{\mathrm{th}} \qquad 71$$

In a similar work, Liu and Zhu (2006) [105] assumed mould copper plate should exhibit thermoelastic behavior and thermoelastic-plastic behaviour for strand. The isotropic linear elastic stress–strain relation was expressed by the constitutive equation as follows:

$$\sigma_{ij} = 2G\varepsilon_{ij} + [\lambda\varepsilon_{kk} - (3\lambda + 2G)\alpha\Delta T]\delta_{ij} \qquad 72$$

It was reported that the total strain can be expressed as the sum of an elastic strain, a thermal strain, and a plastic strain as follows;

$$\varepsilon_{ij} = \varepsilon_{ij}^{\mathrm{e}} + \varepsilon_{ij}^{\mathrm{p}} + \varepsilon_{ij}^{\mathrm{T}} \qquad 73$$

where, temperature change ΔT may induce a thermal strain of a magnitude

$$\varepsilon_{ij}^{\mathrm{T}} = \alpha\Delta T\delta_{ij} \qquad 74$$

In a recent work, Li et al. (2017) [106] reported that in the mushy zone, the stress in solid steel is supposed to increase linearly with the rise in solid fraction between zero strength temperature ($T_{ZST}$) at $f_s$*=0.75 and* zero Xdeformation temperature ($T_{ZDT}$) and it can be given as follows;

$$\sigma_{\mathrm{S}} * \frac{(f_{\mathrm{S}} - f_{\mathrm{ZST}})}{1 - f_{\mathrm{ZST}}} \qquad 75$$

where $f_s$ is solid fraction stress, $f_{zst}$ is stress at zero strength temperature.

Several authors have predicted the probability of crack formation in solid strand by crack susceptibility coefficient SC as follows [104][104], [107], [108];

$$\begin{aligned} S_C \quad &= \frac{Y_M}{Y_C} \ \text{for} \ {}^{T}f_S \le f_S < 1 \qquad 75 \\ &= 0 \ \text{ for } \ 0 \le f_S < T_S \\ &= 0 \ \text{ for } \ Y_M \le 0 \end{aligned}$$

Fengming Du et,. al., (2018) [68]. They studied full model to study complex behavior in vertical slab casting in continuous casting process. They applied inverse algorithm to calculate heat flux and combined with temperature which were measured from thermocouple which placed in casting at different locations to get temperature profile. For evaluating heat flux between mold and slab inverse algorithm used in which heat flux adjusted to match actual temperature taken from thermocouple so that both temperature are within acceptable limit of accuracy. The total strain is sum of elastic strain, plastic and thermal strain.

$$\Delta\{\varepsilon\} = \Delta\{\varepsilon\}_{\mathrm{e}} + \Delta\{\varepsilon\}_{\mathrm{p}} + \Delta\{\varepsilon\}_{\mathrm{T}}(\mathrm{XX}) \qquad 76$$

where $\varepsilon_e$ elastics strain, $\varepsilon_p$ Plastic Strain, $\varepsilon_T$ Thermal strain. Thermal strain is given by

$$\Delta\{\varepsilon\}_{\mathrm{T}} = \left(\{\alpha\} + d[\frac{D]_{\mathrm{c}}^{-1}\{\sigma\}}{dT}\right) dT \qquad 77$$

where α indicates coefficient of thermal expansion. Further in elastic region stress given by

(XX)

$$\Delta\{\sigma\} = [D]_{\mathrm{e}}(\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_{\mathrm{T}}) \qquad 78$$

where $D_e$ Indicates Elastic-Plastic matrix. σ Indicates stress. Further in the plastic region the stress is given by

$$\Delta\{\sigma\} = [D]_{\mathrm{ep}}\ (\Delta\{\varepsilon\} - \Delta\{\varepsilon\}_{\mathrm{T}}) + \Delta\{\sigma\}_{\mathrm{T}} \qquad 79$$

Hence the thermal stress is

$$\Delta\{\sigma\}_{\mathrm{T}} = \frac{[D]_c((\partial\bar{\sigma}\,\partial H)/(\partial\{\sigma\}\,\partial T))dT}{H' + \{\sigma\bar{\sigma}/\partial\{\sigma\}\}^{\mathrm{T}}[D_{ec}(\partial\bar{\sigma}/\partial\{\sigma\})} \qquad 80$$

where $\bar{\sigma}$ indicates equivalent stress at node.

They noted that near the meniscus liquid fraction is more compared to bottom slab. It shows that solidification is start early at bottom side. Because of uneven temperature in slab leads to thermal strain which creates thermal stress.

Hadata et al., [11] studied surface crack defect evaluation four criteria used namely plastic work criteria, Rice and Tracy Criteria, modified Rice and Tracy criteria and Latham criteria. Plastic work criteria can be given by following equation

$$C_{EP} = \int_0^1 \bar{\varepsilon}\bar{\sigma}dt \ \text{ for } \ \sigma_m > 0 \qquad 81$$

where ε indicates strain rate, σ indicates stress. This criteria based on assumption that crack will get generated if strain energy is more than critical value $C_{EP}$. Plastic strain is evaluated only in region where mean stress is positive.

Following is the criteria given by Rice & Tracy

$$C_{RT} = \bar{\varepsilon}\exp\left(-\frac{3}{2}\frac{6m}{\bar{\sigma}}\right) \qquad 82$$

where $\sigma_m$ is mean stress & ε indicates strain. This criteria assumes that crack will appear if strain increases beyond $C_{RT}$.

Following is the criteria given by modified which uses only positive values of strain for calculation of critical parameter $C_{RM}$. The

$$C_{RM} = \sum\Delta\bar{\varepsilon}\exp\left(-\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\right) \text{ for } \sigma_m > 0 \qquad 83$$

For Latham Criteria equation is as follows

$$C_{LO} = \int_0^t \sigma_{\max}\dot{\varepsilon}\ \mathrm{dt} \text{ for } \sigma_m > 0 \qquad 84$$

This criterion uses only positive values of maximum stress $\sigma_{max}$. it states that crack will form once strain work done by maximum given tensile stress exceeds critical value. Fachinotti et al. (2006) [102] to study the macro-segregation defects in strand caused by thermal stress. Because of cooling at surface when metal is solidified metal goes under compression at the same time at solidification front tension is generated.

Study Carried by Alizadeh et al. [3]. They compared FPM results with FVM results. It was concluded that heat transfer, surface temperature, and shell thickness can be successfully modeled by FPM method also known as Quadratic Upwind Interpolation (QUICK). For solidification enthalpy porosity model they used along with transient energy equation and solved using finite volume method. This model is capable to predict interfacial gap between shell and mold. They observed that casting speed is most critical parameter in controlling solidified shell thickness which indicates stability of dimension after cooling and its accuracy.

Many literatures have reported about strand bulging between rolls which have caused transverse cracks, radial streaks and centerline macrosegregation [9], [10], [102]. Risso et al. [109] evaluated the thermal stress and strain in the solidifying shell of the strand by using the analytical method. Recently, Chen et al. (2019) [110] investigated the mold level fluctuations. These fluctuations are caused by transient bulging of the solidifying shell. Consequently, transient bulging phenomenon affects the quality of the steel. They developed a 1D and 2D model for strand simulation. They reported that mold level fluctuations are highly caused by dynamic bulging. Several constitutive models have been adopted for simulating the solidification stresses using the simple elastic-plastic models [111][112]. Researchers added a separate creep model for transient modeling [113]. The integration of these transient constitutive laws and further, mathematical modeling is a challenging task. From all the above discussion it is observed that the temperature and stress-strain distribution in the strand region of the continuous casting process plays an important role in defining the quality of the final solidified product[29], [114]–[116]. In 2006, Liu and Zhu [105] developed a three-dimensional finite-element heat-transfer and thermal stress models to study the thermo-mechanical distortion on the slab during operation. They reported that operating parameters i.e., casting affected the strand distortion in copper walls of the mould. Pascon and coworkers (2006) [117] studied the generation of transverse crack during bending and straightening of strands. The numerical model was applied and validated with industrial data. The transverse cracks were found at the upper face of the strand. A numerical model was presented by Fachinotti et al. (2006) [102]to study the macro-segregation defects in strand caused by thermal stress. They made a hypothesis about the transient effect of alternate rolling and bulging. To measure surface temperature and shell thickness, finite point method was used by Alizadeh et al. [3]. They compared FPM results with FVM results. It was concluded that heat transfer, surface temperature, and shell thickness can be successfully modeled by FPM method.

## 6. Electromagnetic Stirring in Mould

The application of magnetohydrodynamics (MHD) techniques in the continuous casting process started to improve steel quality and increase the process capability. The use of MHD technology in metallurgical processes is rapidly advancing [118]. The application of electromagnetic forces in metallurgical processes is for melt stirring, electromagnetic braking, flow control of melt, meniscus stabilization and enhancing the solidification properties as well as for quality improvement. Many researchers have studied the melt flow behavior in the ladle. The swirl motion and vortex generation in ladle have a significant impact on the quality of steel. In a recent application, the ladle is stirred by the use of MHD forces. The impact of heat generation is studied when MHD is used in the ladle. The MHD has found wide application in the area of continuous casting molds. The mold has a complex molten metal flow structure. The flow in continuous casting mold is unstable and transient. The incoming jet from submerged entry nozzle interacts with the free surface of melt in mold consequently it creates instability in melt and chaotic behavior flow. Further, molten steel flow behavior becomes highly turbulent causing large meniscus level fluctuations. Subsequently, high fluctuation of meniscus level causes entrapment of inclusions in the solidified shell [119]. The application of MHD force is an attractive method to control the molten steel flow in the mold, where physical interference is not possible due to high temperature.[120]

Magnetohydrodynamics (MHD) refers to the study of the magnetic properties of electrically conducting fluids. The Lorentz force is generated to stir the molten metal in the EMS system. The induction equations can be written as follows [86]:

$$\frac{\partial\vec{\beta}}{\partial t}+(\vec{U}\cdot\nabla)\vec{B}=\frac{1}{\mu\sigma}\nabla^2\vec{B}+(\vec{B}\cdot\nabla)\vec{U} \qquad 85$$

The magnetic field $\overrightarrow{B_0}$ was applied externally. The external magnetic field develops magnetic field $\vec{b}$ in liquid metal steel in mould. Therefore, only induced magnetic field $\vec{b}$ is solved through numerical solution. The solution gives the total value of magnetic filed. required to be solved to get the total magnetic field $\overrightarrow{B_0}$ and $\vec{b}$. The magnetic field generation is expressed as [86]:

$$\frac{\partial\vec{b}}{\partial t}+(\vec{U}\cdot\nabla)\vec{b}=\frac{1}{\mu\sigma}\nabla^2\vec{b}+\left[\left(\vec{B}+\vec{b}\right)\cdot\nabla\right]\vec{U}-(\vec{U}\cdot\nabla)\vec{B} \qquad 86$$

Further, the Lorentz force $\vec{F}$ generated in the lqiquid metal fluid can be explained as below. The Lorentz force is complimented to the momentum equation as follows;

$$\vec{F}=\vec{J}X\,\vec{B}=\vec{J}\,X\left(\vec{B}+\vec{b}\right) \qquad 87$$

Heat transfer and solidification equations can be written as follows. For solidification process, the energy conservation equation is given as[86]:

$$\rho\frac{\partial H}{\partial t}+\rho\nabla\cdot\overrightarrow{(U}\mathrm{H})=\nabla\cdot(k_{eff}\nabla T)+Q_L \qquad 88$$

where, $k_{eff}$ is effective thermal conductivity.

The solidified shell formed is pulled out at a constant casting velocity, $\vec{U}_{pull}$. Hence, the region having liquid fraction (*f*) equal to zero will move along the casting direction with the casting speed. The liquid fraction can be calculated as follows[86]:

$$f_1=\begin{cases}0\\1\end{cases}\frac{T-T_{solidus}}{T_{liquidus}-T_{solidus}} \qquad 89$$

Mass conservation of melt in the metallurgical process can be given as[121]:

$$\nabla\cdot\vec{u}=S_{shell,mass} \qquad 90$$

where $\vec{u}$ is velocity in the three coordinate directions, and $S_{shell;mass}$ is a mass sink term to account for solidification of the molten steel. Time dependent momentum balance equation can be written as follows:

$$\frac{\partial}{\partial t}(\rho u_i)+\frac{\partial}{\partial x_j}(\rho u_i u_j)=-\frac{\partial p^*}{\partial x_j}+\frac{\partial}{\partial x_j}\left[(u+u_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right]+\vec{F} \qquad 91$$

$p^*$ is modified pressure $p^*+\frac{2}{3}\rho k_r$, $p$ is the gauge pressure, $k_r$ is residual kinetic energy.

Some assumptions were made by researchers to reduce the complexity during the numerical modeling of electromagnetic processing of materials. The molten steel is assumed as homogeneous. Also, molten steel is considered as Newtonian incompressible fluid with constant thermo-mechanical properties. Moreover, the boundary condition for the induced magnetic field can be assumed to be tangential to the wall. Further, it was reported that electromagnetic stirring generates a magnetic field which can be assumed as homogenous magnetic mold.[122][123] Apart from the finite volume method, many researchers have solved the electromagnetic equation by finite element method.

Electromagnetic stirring (EMS) of molten metal has become an important part of the continuous casting operation.[124], [125] EMS improves the quality of semi-finished sections by electromagnetically stirring the crystallizing molten steel during solidification. The EMS has lots of advantages, it affects the heat and mass transfer of molten steel during the solidification. In 2004, Chang, Hull, and Beitelmann [126] performed a numerical simulation a novel electromagnetic stirring system that employs two rotating magnetic fields in a continuous casting mold. The model included two magnetic field zone. The upper part controlled the stirring flow in the meniscus region and the remaining part was under the influence of an electromagnetic stirring (M-EMS) system.

In 2004, Cramer et al. [127] conducted a laboratory investigation using liquid metals having low melting temperature than 300°c. They examined an aluminum alloy investment casting process. The high flow velocities were controlled by the application of a static magnetic field. The local velocity measurements, as well as integrated flow rate, were calculated by using eutectic InGaSn ($T_{melt}$ = 10°C).

In 2007, Eckert et al. [128] investigated electromagnetic stirring of molten metal in mould. The liquid metal was used as fluid medium to mimic the molten steel. The liquid metal has low melting temperature, thus a good option for experimental work. The experiment was carried by UDV instruments. In this investigation, they developed some instruments to investigate the flow measurements. Further, they used alternate stirring method by using a rotating magnetic field (RMF).

One of the purposes of M-EMS is to enhance to break the coarse columnar dendritic solidification structure. This would lead to yield a finer dendritic structure. These structure may be like equiaxed grains in larger area. In another approach, Javurek et al. (2008) [123] modeled an in-mold electromagnetic stirring (M-EMS) for the study. They used the physical setup for PIV measurements and further studied the turbulent flow. They also investigated the other parameters like steel temperature, solidification of liquid metal, inclusion movement in liquid and electromagnetic force influence using numerical formulation. In this simulation they considered the affect of the magnetic field on the liquid metal. The resulting radial, respective tangential force densities are;

$$\bar{F}_r = -\frac{1}{8}B_0^2\left(\omega - \frac{u_t}{r}\right)^2\sigma^2\mu_M r^3 \tag{92}$$

$$\bar{F}_t = \frac{1}{2}B_0^2\left(\omega - \frac{u_t}{r}\right)\sigma r \tag{93}$$

In above equation, $B_0$ is the magnitude of magnetic field and omega is angular velocity. The EM properties of liquid metal are expressed as σ.

The electromagnetic stirring (EMS) of liquid metal in the continuous-casting mold becomes an essential process. The magnetic devices are installed in the mold, at different zones depending on the requirements. In 2009, Sivak et al. [124] have investigated the effect of electromagnetic stirring of the liquid phase of the crystallizing ingot in the mold. Subsequently, the quality of semifinished sections and rounds cast on semi-continuous casting machines were studied. It was reported that EMS units were developed for the use in molds for the efficient stirring of the liquid phase. Further investigations on wavy meniscus profile in billet continuous casting mold have been studied by M. Cho, Park, and Kim (2010) [129]. They developed an electromagnetic shield to improve the transient unstable meniscus profile with in-mold electromagnetic stirring (M-EMS).

Vogl et al. (2013) [130] developed a numerical model to simulate the gas bubbles behavior under the presence of magnetic fields. They studied the argon bubble entrapment phenomenon and further it was observed that the stirring can intensify the instabilities in melt flow. The low centered macrosegregation in high carbon steel is highly desirable. In 2015, Zeng et al. [131] studied the final permanent magnet stirring (FPMS) method to produce steel with low macro segregation. They developed a three-dimensional unsteady numerical model to analyze the magnetic flux density, the electromagnetic force, and the molten steel flow. The numerical model was based on electromagnetic field analysis software Opera-3D and the flow field software Fluent. It was found that magnetic flux density is nearly invariable. It was recommended that stirring frequency should be increased to obtain uniform core elements. Further, it was claimed that FPMS is an effective technology to improve center segregation. It was concluded that FPMS improved the solidification microstructure and reduced the wire rod segregation.

The quality of steel billet can be sufficiently improved by the use of mold electromagnetic stirrers

(M-EMS). Recent research of An et al. (2017) [132] focuses on the finite element based numerical modeling of the electromagnetic field, flow field, and temperature distribution in the mold. It was found that an increase in current develops an unstable flow field. In contrast, the effect of the frequency of the flow field has a greater impact. Further, decreasing the frequency affects the electromagnetic field, flow field, temperature distribution and solidification of molten steel in the mold.

In 2017, Maurya and Jha [86] numerically investigated the effect of stirrer position on melt flow and further, solidification in a continuous casting billet mold under the influence of MHD forces. It was reported that recirculation loops were observed above and below the stirrer position when the magnetic field was applied. Moreover, the size of the recirculation loop was seen decreased when solidification equations were solved for the entire domain. Zhang et al. (2019) [133] studied the electromagnetic stirring in the mold to evaluate the macroscopic transport phenomenon.

In 2019, Schurmann et al.[134] studied the electromagnetic stirring of mold and further, the effect of swirling nozzle by using liquid GaInSn at Mini-LIMMCAST facility. The investigation was focused on the interaction between the flow driven by the GYRONOZZLE and electromagnetic stirring in the mold. Recently, Zhang et al. (2019) [133] reported that magnetic field produced the horizontal swirling flow of melt in a continuous casting mold.

## 7. Coupled FVM and FEM approach

To understand the complete phenomenon in continuous casting systems, molten steel flow, and thermal models are coupled with thermal distortion model (structural deformation) models. Several authors reported transient modeling of molten steel flow and heat transfer in the mold using various tool and techniques[135], [136][137]. Very few literatures are available till date on coupled analysis. In previous cases of coupled study[44], [106], [138], the different numerical methodology was used to couple melt flow, solidification, and deformation in strand. Hwan et al. (1999) [101]carried a detailed study on coupled analysis of molten steel flow, heat transfer, and deformation in strand. The numerical model was able to predict the crack formation area and significant parameters affecting the strain propagation. Lee et al., (2000) [104] investigated crack formation in a continuously cast steel beam. They developed a coupled numerical model (FEM and FVM) to analyses the molten steel flow, heat transfer and deformation in a solidifying strand.

In 2007, Shamsi and Ajmani [40]carried a numerical investigation on a strand which had a length of 30 m. Further, Seyedein and Hasan [139][140] investigated a slab casted strand up to three meters. Generally, in this type of numerical modeling several assumptions are made for example as geometry simplification, boundary conditions, thermo-mechanical properties of mold and steel, etc. Most of the researchers have considered constant values of thermo-mechanical properties and boundary conditions. Seyedein and Hasan [139][140] used a constant value of strand cooling as 750 $Wm^{-2}K^{-1}$. Moreover, using symmetrical boundary conditions are also not suitable with some turbulence model due to asymmetrical flows in the mold region [94].

In 2003, Ho-Mun et al. [141]proposed a hybrid model of finite element and finite difference to calculate thermal strain. Recently, Svensson et al. [45](2015) developed a coupled model of CFD-CC model with structural Finite Element Analysis (FEA) to predict stress-strains as a function of irregular lubrication conditions in the mold. The solidified shell structure and mesh was exported to the FE model. It was reported that the temperature distribution within the shell causes shrinkage and thermal deformation; which are in turn, the main source of stress. The temperature dependent material properties, transient modeling of has been not reported by any author using FVM and FVM coupled approach. Furthermore, the quality of the solidified strand depends upon operating parameters of the continuous casting process. None has reported the parametric analysis of operating parameters such as liquid temperature, cooling rate, and casting speed on the structural deformation using a coupled model approach.

In previous cases of coupled study, the different numerical methodology was used to couple melt flow, solidification, and deformation in strand. Some authors report only limited coupled investigations using heat molten steel flow and heat transfer study. In some cases, FDM and FEM approach was used. While very few authors investigated using coupling FVM and FEM approach. The coupled analysis has added advantage over other conditions. It gives a better understanding of investigations because of the continuous use of data in numerical modeling of fluid flow, heat transfer, solidification, and thermo-mechanical deformation. Moreover, thermo-mechanical properties of steel are varied with respect to temperature. Especially thermo-mechanical properties change significantly during solidification and phase transformation. Strictly speaking, temperature dependent material properties, compressible transient modeling of has been not reported by any author using FVM and FVM coupled approach. Furthermore, the quality of the solidified strand depends upon operating parameters of the continuous casting process. Any alteration in standard optimized operation parameters will have an adverse effect because of the rapid cooling nature of strand.

## 8. Summary

Numerical modeling of continuous casting mold and strand has been done by several researchers since the last three decades[36], [38], [142]–[144]. Previous research work can be classified in three broad categories namely, numerical work on melt flow[133], [145]–[147] and heat transfer in mold [114], [148], [149], modeling of cooling and thermo-mechanical study of strand[39], [43], [150] and coupled numerical modeling of melt flow[45], [106]. The early work started with numerical modeling of molten steel flow and heat transfer in mold later thermo-mechanical studies prevailed. The numerical models have been validated from analytical, experimental or industrial plant data[151]. Till date, the basic heat transfer model is used for online control and operation in the industry[151]. However, these validations of the numerical model are available for limited cases and conditions. Most of the earlier investigations on mold were based on steady-state modeling. However, several authors reported transient modeling of molten steel flow and heat transfer in the mold using various tool and techniques[152], [153]. However, very few literatures are available till date on coupled analysis[115][154]. The coupled analysis gives a complete insight of melt flow in mold, temperature distribution and thermo-mechanical deformation in continuous casting strand. In previous cases of coupled study, the different numerical methodology was used to couple melt flow, solidification, and deformation in strand. Some authors report only limited coupled investigations using heat molten steel flow and heat transfer study[98], [106]. In some cases, FDM and FEM approach was used[115], [141]. While very few authors investigated using coupling FVM and FEM approach. The coupled analysis has added advantage over other conditions. It gives a better understanding of investigations because of the continuous use of data in numerical modeling of fluid flow, heat transfer, solidification, and thermo-mechanical deformation.

Moreover, thermo-mechanical properties of steel are varied with respect to temperature. Especially thermo-mechanical properties change significantly during solidification and phase transformation. It is mentioned in the assumptions of many researchers that linear thermo-mechanical properties are used so as to reduce complexity in mathematical modelling[49][83][155][156]. Some thermo-mechanical properties vary with respect to temperature by considerable amount such as thermal conductivity, viscosity etc. As molten metal is at high temperature its thermal conductivity will increase as it undergoes cooling process. An equivalent thermal conductivity is considered for simulating the convection heat flow of liquid and mushy zones[157]. Heat transfer through convection will decrease as the cooling air surrounding casting will heat and which will decrease thermal gradient, hence rate of heat transfer. Researchers used effective thermal conductivity to consider this effect. Heat transfer models based on the effective thermal conductivity method are unable to reliably investigate mold heat transfer even though they can be used to determine the end of the liquid pool[33][51][81][94]. This necessitates consideration of nonlinear thermo-mechanical properties with due consideration in heat transfer during solidification process. A research carried out using nonlinear thermo-mechanical properties and had used 2D model, in which thermal deformation in third dimension is not taken in account such as bulging effect in CC process[136]. Those issues can be captured with consideration of 3D model.

Most of researchers used enthalpy porosity approach for numerical modeling and which were unable to find accurate prediction of surface temperature[158][94]. During solidification heat transfer is nonlinear in nature and complex phenomena. The solidified layer has a thickness non-uniform in the length of continuous cast slab, which is caused by fluid movement[52]. Using the simplified model of heat transfer does not notice this phenomenon because of error in accurately predicting surface temperature in casting. There are other effects such as cracks on casting surface, bulging, non-uniform thickness which are arising because of inaccurate prediction of surface temperature in casting[102]. Using nonlinear material properties for heat transfer and cooling will help to accurately predict surface temp in solidification process.

It can be concluded that the model with turbulence gives much larger variation of the meniscus geometry and higher temperature in the top part of the strand region[53]. Researches used effective viscosity instead of non-linear nature to avoid complications in mathematical modelling[50][151][105]. Turbulence model is accurate as the turbulent viscosity plays significant role in casting process, using nonlinear properties further improvement is expected. The developed numerical model of the continuous casting uses experimentally measured values of the material properties[98]. In that case, the simulations based on the model allow to determine the stress and strain distribution with a higher accuracy. It can help to predict the optimal conditions for the continuous casting of steel. Thus, use of nonlinear material properties can be used to improve CC Process and to find optimum CC process parameters, which lead to accurate modelling of thermal distortion and residual stresses in casting.

In addition to this, composition of steel has significant impact on thermo-mechanical properties. An investigation related to different compositions could bring more information on structural deformation. These considerations have been discretely taken by authors in numerical modeling. However, temperature dependent material properties, transient modeling of has been not reported by any author using FVM and FVM coupled approach. Furthermore, the quality of the solidified strand depends upon operating parameters of the continuous casting process. Any alteration in standard optimized operation parameters will have an adverse effect because of the rapid cooling nature of strand. None has reported the parametric analysis of operating parameters such as liquid temperature, cooling rate, and casting speed on the structural deformation using a coupled model approach. The structural deformation and defect occurring in a strand is a complex phenomenon based on heat transfer and metallurgical properties of steel. Apart from it, surface defects are very frequent in solidified strands. Surface defects arises from several mechanical forces working on strand such as ferro-static pressure, roller misplacement and design[151]. Till date, numerical modeling considering metallurgical approach for surface defect, segregation defect and crack formation remains a big challenge.

## Funding

This study has not any external/internal funding source.

## References

[1] World Steel Association, “World Steel in Figures 2017,” 2017.

[2] H. Precht and T. Preston, “Continuous Casting of Steel Slabs,” in *SAE Technical Paper Series*, 2010, vol. 1, p. 207.

[3] M. Alizadeh, S. A. J. Jahromi, and S. B. Nasihatkon, “Applying Finite Point Method in Solidification Modeling during Continuous Casting Process,” *ISIJ Int.*, vol. 50, no. 3, pp. 411–417, 2010.

[4] W. R. Irving, *Continuous Casting of Steel.* Institute of Materials, 1993.

[5] D. D. Geleta, M. I. H. Siddiqui, and J. Lee, “Characterization of Slag Flow in Fixed Packed Bed of Coke Particles,” *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 51, no. 1, pp. 102–113, Feb. 2020.

[6] M. I. H. Siddiqui and P. K. Jha, “Numercal Investigation of Grade Intermixing and Heat Transfer during Ladle Change-Over in Steelmaking Tundish,” in *23rd International Conference on Processing and Fabrication of Advanced Materials, IIT Roorkee*, 2014, pp. 981–993.

[7] M. I. H. Siddiqui and M. H. Kim, “Optimization of flow control devices to minimize the grade mixing in steelmaking tundish,” *J. Mech. Sci. Technol.*, vol. 32, no. 7, pp. 3213–3221, 2018.

[8] M. I. H. Siddiqui and P. K. Jha, “Effect of Tundish Shape on Wall Shear Stress in a Multi-strand Steelmaking Tundish,” in *International Conference on Smart Technologies for Mechanical Engineering*, 2013, vol. 86, no. 9999, pp. 122–130.

[9] B. G. Thomas, “Modeling of the continuous casting of steel—past, present, and future,” in *metallurgical and materials transactions B*, 2002, vol. 33, no. 6, pp. 3–30.

[10] M. Bellet and A. Heinrich, “A Two-dimensional Finite Element Thermomechanical Approach to a Global Stress-Strain Analysis of Steel Continuous Casting,” *ISIJ Int.*, vol. 44, no. 10, pp. 1686–1695, 2008.

[11] C. Li and B. G. Thomas, “Maximum casting speed for continuous cast steel billets based on sub-mold bulging computation,” *85th Steelmak. Conf. proceedings, ISS*, pp. 109–130, 2002.

[12] B. G. Thomas, “Issues in Thermal-Mechanical Modeling of Casting Processes.,” *ISIJ Int.*, vol. 35, no. 6, pp. 737–743, 2008.

[13] K. N. Seetharamu, R. Paragasam, G. A. Quadir, Z. A. Zainal, B. S. Prasad, and T. Sundararajan, “Finite element modelling of solidification phenomena,” *Sadhana - Acad. Proc. Eng. Sci.*, vol. 26, no. 1–2, pp. 103–120, Feb. 2001.

[14] M. B. N. Shaikh, M. Alam, and M. I. H. Siddiqui, “Application of Electromagnetic Forces in Continuous Casting Mold: A Review,” *Int. J. Adv. Prod. Mech. Eng.*, vol. 2, no. 5, pp. 43–49, 2016.

[15] M. Alam, T. Q. Hashmi, and M. I. H. Siddiqui, “Effect of shroud depth and advance pouring box on fluid flow and inclusion floatation behaviour in a slab caster steelmaking tundish,” *J. Mater. Sci. Mech. Eng.*, vol. 2, no. 22, pp. 1941–1945, 2015.

[16] M. I. H. Siddiqui, “Investigation of Flow Behaviour and Inclusion Removal Mechanism in a Multi-Strand Tundish With Strand Blockages,” Indian Institute of Technology Roorkee, 2011.

[17] M. I. H. Siddiqui, P. K. Jha, and S. A., “Effect of Molten Steel Inflow Rate on Grade Mixing in Tundish,” in *National Conference on Mechanical Engineering Ideas, Innovation*

*and Initiatives*, 2016, vol. 1, no. 1, p. 221.

[18] M. I. H. Siddiqui, A. Maurya, F. Asiri, and R. Kumar, "Mathematical modeling of continuous casting tundish-A Review," *VW Appl. Sci.*, vol. 3, no. 1, pp. 92–103, 2021.

[19] R. Kumar, M. I. H. Siddiqui, and P. K. Jha, "Numerical Investigations on the use of Flow Modifiers in Multi-Strand Continuous Casting Tundish using RTD Curves Analysis," in *Proceedings of STEM-2013 International Conference on Smart Technology for Mechanical Engineers*, 2013, no. October, pp. 603–612.

[20] C. Li and B. G. Thomas, "Thermomechanical finite-element model of shell behavior in continuous casting of steel," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 35, no. 6, pp. 1151–1172, Dec. 2004.

[21] M. I. H. Siddiqui and P. K. Jha, "Modeling of Molten Steel Interface and Grade Mixing in a Tundish Using VOF Model," in *Proceedings of the 22th National and 11th International ISHMT-ASME Heat and Mass Transfer Conference*, 2013.

[22] M. I. H. Siddiqui, D. D. Geleta, G. Bae, and J. Lee, "Numerical Modeling of the Inclusion Behavior during AC Flash Butt Welding," *ISIJ Int.*, vol. 60, no. 11, pp. 1–9, 2020.

[23] M. I. H. Siddiqui and M.-H. H. Kim, "Optimization of flow control devices to minimize the grade mixing in steelmaking tundish," *J. Mech. Sci. Technol.*, vol. 32, no. 7, pp. 3213–3221, 2018.

[24] R. Kumar, A. Maurya, M. I. H. Siddiqui, and P. K. Jha, "Some studies in diffrent shapes of tundish-intermixing and flow behaviour," in *4th International Conference on Production & Industrial Engineering*, 2016.

[25] P. K. J. M.I.H. Siddiqui, "Assessment of turbulence models for prediction of intermixed amount with free surface variation using coupled level set volume of fluid method," *ISIJ Int.*, vol. 54, no. 11, p. 2578, 2014.

[26] W. Ahmad and M. I. H. Siddiqui, "Study of Grade Intermixing and Heat Transfer in Two Different Shapes of Tundishes," in *Processing and Fabrication of Advanced Materials : XXIII*, 2014, vol. 2, pp. 994–1009.

[27] M. I. H. Siddiqui and P. K. Jha, "Effect of Inflow Rate Variation on Intermixing in a Steelmaking Tundish During Ladle Change-Over," *Steel Res. Int.*, vol. 87, no. 6, pp. 733–744, 2016.

[28] M. I. H. Siddiqui and M.-H. Kim, "Two-Phase Numerical Modeling of Grade Intermixing in a Steelmaking Tundish," *Metals (Basel).*, vol. 9, no. 1, p. 40, 2019.

[29] S. Koric, L. C. Hibbeler, and B. G. Thomas, "Explicit coupled thermo-mechanical finite element model of steel solidification," *Int. J. Numer. Methods Eng.*, vol. 78, no. 1, pp. 1–31, Apr. 2009.

[30] Y. Yin, J. Zhang, Q. Dong, and Q. H. Zhou, "Review on Modeling and Simulation of Continuous Casting," *Ironmak. Steelmak.*, vol. 46, no. 9, pp. 855–864, 2019.

[31] X. Huang and B. G. Thomas, "Intermixing model of continuous casting during a grade transition," *Metall. Mater. Trans. B*, vol. 27, no. 4, pp. 617–632, Apr. 1996.

[32] B. Petrus, K. Zheng, X. Zhou, B. G. Thomas, and J. Bentsman, "Real-time, model-based spray-cooling control system for steel continuous casting," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 42, no. 1, pp. 87–103, Feb. 2011.

[33] B. G. Thomas and L. Zhang, "Mathematical Modeling of Fluid Flow in Continuous Casting," *Rev. Lit. Arts Am.*, vol. 41, no. 10, pp. 1181–1193, 2001.

[34] Y. Meng and B. G. Thomas, "Modeling Transient Slag-Layer Phenomena in the Shell/mold Gap in Continuous Casting of Steel," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 34, no. 5, pp. 707–725, 2003.

[35] J. Mahmoudi, "Mathematical modelling of fluid flow, heat transfer and solidification in a strip continuous casting process," *Int. J. Cast Met. Res.*, vol. 19, no. 4, pp. 223–236, 2006.

[36] Z.-D. Qian and Y.-L. Wu, "Large Eddy Simulation of Turbulent Flow with the Effects of DC Magnetic Field and Vortex Brake Application in Continuous Casting," *ISIJ Int.*, vol. 44, no. 1, pp. 100–107, 2004.

[37] J. K. Park, B. G. Thomas, I. V. Samarasekera, and S. U. Yoon, "Thermal and mechanical behavior of copper molds during thin-slab casting (I): Plant trial and mathematical modeling," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 33, no. 3, pp. 425–436, 2002.

[38] S. Mazumdar and S. K. Ray, "Solidification control in continuous casting of steel," *Sadhana*, vol. 26, no. 1–2, pp. 179–198, 2001.

[39] B. Hadała, A. Cebo-Rudnicka, Z. Malinowski, and A. Gołdasz, "The Influence of Thermal Stresses and Strand Bending on Surface Defects Formation in Continuously Cast Strands," *Arch. Metall. Mater.*, vol. 56, no. 2, pp. 367–377, 2011.

[40] M. R. R. I. Shamsi and S. K. Ajmani, "Three Dimensional Turbulent Fluid Flow and Heat Transfer Mathematical Model for the Analysis of a Continuous Slab Caster," *ISIJ Int.*, vol. 47, no. 3, pp. 433–442, 2007.

[41] Y. Sahai, "Tundish Technology for Casting Clean Steel: A Review," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 47, no. 4, pp. 2095–2106, 2016.

[42] A. E. Huespe, A. Cardona, and V. Fachinotti, "Thermomechanical model of a continuous casting process," *Comput. Methods Appl. Mech. Eng.*, vol. 182, no. 3–4, pp. 439–455, 2000.

[43] J. X. Fu, W. S. Hwang, J. S. Li, S. F. Yang, and Z. Hui, "Effect of casting speed on slab broadening in continuous casting," *Steel Res. Int.*, vol. 82, no. 11, pp. 1266–1272, 2011.

[44] J. R. Boehmer, G. Funk, M. Jordan, and F. N. Fett, "Strategies for coupled analysis of thermal strain history during continuous solidification processes," *Adv. Eng. Softw.*, vol. 29, no. 7–9, pp. 679–697, Aug. 1998.

[45] J. Svensson, P. E. R. López, P. N. Jalali, and M. Cervantes, "One-way coupling of an advanced CFD multi-physics model to FEA for predicting stress-strain in the solidifying shell during continuous casting of steel," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 84, no. 1, 2015.

[46] B.-Z. Ren, D.-F. Chen, H.-D. Wang, M.-J. Long, and Z.-W. Han, "Numerical simulation of fluid flow and solidification in bloom continuous casting mould with electromagnetic stirring," *Ironmak. Steelmak.*, vol. 42, no. 6, pp. 401–408, 2015.

[47] X. Jin*, D. F. Chen, D. J. Zhang, X. Xie, Y. Y. Bi, and X. Jin, "Water model study on fluid flow in slab continuous casting mould with solidified shell," *Ironmak. Steelmak.*, vol. 38, no. 2, pp. 155–159, Feb. 2011.

[48] H. K. Versteeg and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, 2nd-ed ed. Pearson, Prentice Hall, 2007.

[49] B. Zhao, B. G. Thomas, S. P. Vanka, and R. J. O'Malley, "Transient Flow and Temperature Transport in Continuous Casting of Steel Slabs," *J. Heat Transfer*, vol. 127, no. 8, p. 807, 2005.

[50] B. Li and F. Tsukihashi, "Vortexing Flow Patterns in a Water Model of Slab Continuous Casting Mold," *ISIJ Int.*, vol. 45, no. 1, pp. 30–36, 2005.

[51] L. Sowa and A. Bokota, "Numerical model of thermal and flow phenomena the process growing of the CC slab," *Arch. Metall. Mater.*, vol. 56, no. 2, pp. 359–366, 2011.

[52] M. H. H. Zare, a. H. H. Meysami, S. Mahmoudi, M. Hajisafari, and M. Mazar Atabaki, "Simulation of flow field and steel/slag interface in the mold region of a thin slab steel continuous caster with tetra-furcated nozzle," *J. Manuf. Process.*, vol. 15, no. 3, pp. 307–317, Aug. 2013.

[53] W. Chen, Y. Ren, L. Zhang, and P. R. Scheller, "Numerical Simulation of Steel and Argon Gas Two-Phase Flow in Continuous Casting Using LES + VOF + DPM Model," *Jom*, vol. 71, no. 3, pp. 1158–1168, 2019.

[54] D. D. Geleta, M. I. H. Siddiqui, and J. Lee, "Characterization of Slag Flow in Fixed Packed Bed of Coke Particles," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 51, no. 1, pp. 102–113, 2020.

[55] M. I. H. Siddiqui and M. H. Kim, "Two-phase numerical modeling of grade intermixing in a steelmaking Tundish," *Metals (Basel).*, vol. 9, no. 1, p. 40, Jan. 2019.

[56] J. R. De Sousa Rocha, E. E. B. De Souza, F. Marcondes, and J. A. De Castro, "Modeling and computational simulation of fluid flow, heat transfer and inclusions trajectories in a tundish of a steel continuous casting machine," *J. Mater. Res. Technol.*, vol. 8, no. 5, pp. 4209–4220, Sep. 2019.

[57] D.-Y. Sheng and Q. Yue, "Modeling of Fluid Flow and Residence-Time Distribution in a Five-Strand Tundish," *Met. 2020, Vol. 10, Page 1084*, vol. 10, no. 8, p. 1084, Aug. 2020.

[58] M. I. H. Siddiqui, D. D. Geleta, G. Bae, and J. Lee, "Numerical Modeling of the Inclusion Behavior during AC Flash Butt Welding," *ISIJ Int.*, vol. 60, no. 11, pp. 2503–2511, 2020.

[59] M. I. H. Siddiqui, H. Alshehri, J. Orfi, M. A. Ali, and D. Dobrota, "Computational Fluid Dynamics (CFD) Simulation of Inclusion Motion under Interfacial Tension in a Flash Welding Process," *Metals (Basel).*, vol. 11, no. 7, p. 1073, 2021.

[60] Y. Ruan, Y. Yao, S. Shen, B. Wang, J. Zhang, and J. Huang, "Physical and Mathematical Simulation of Surface-Free Vortex Formation and Vortex Prevention Design during the End of Casting in Tundish," *steel Res. Int.*, vol. 91, no. 6, p. 1900616, Jun. 2020.

[61] M. A. Saeidy Pour and S. Hassanpour, "Steel Cleanliness Depends on Inflow Turbulence Intensity (in Tundishes and Molds)," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 51, no. 5, pp. 2199–2210, Oct. 2020.

[62] M. I. H. Siddiqui and P. K. Jha, "Assessment of Turbulence Models for Prediction of Intermixed Amount with Free Surface Variation Using Coupled Level-Set Volume of Fluid Method," *ISIJ Int.*, vol. 54, no. 11, pp. 2578–2587, 2014.

[63] Y. Liu, Z. He, and L. Pan, "Numerical Investigations on the Slag Eye in Steel Ladles," *Adv. Mech. Eng.*, vol. 2014, pp. 1–6, 2014.

[64] H.-X. Li, Q. Wang, H. Lei, J.-W. Jiang, Z.-C. Guo, and J.-C. He, "Mechanism Analysis of Free-Surface Vortex Formation during Steel Teeming," *ISIJ Int.*, vol. 54, no. 7, pp. 1592–1600, 2014.

[65] S. K. Choudhary and D. Mazumdar, "Mathematical modelling of fluid flow, heat transfer and solidification phenomena in continuous casting of steel," *Steel Res.*, vol. 66, no. 5, pp. 199–205, May 1995.

[66] J. Szekely and R. T. Yadoya, "The physical and mathematical modelling of the flow field in the mold region in continuous casting systems: Part II. The mathematical representation of the turbulent flow field," *Metall. Trans.*, vol. 4, no. 5, pp. 1379–1388, 1973.

[67] B. G. Thomas, Q. Yuan, S. Sivaramakrishnan, T. Shi, S. P. Vanka, and M. B. Assar,

"Mathematical Modeling of Iron and Steel Making Processes. Comparison of Four Methods to Evaluate Fluid Velocities in a Continuous Slab Casting Mold.," *ISIJ Int.*, vol. 41, no. 10, pp. 1262–1271, 2008.

[68] B. Launder and D. Spalding, "The Numerical Computation of Turbulent Flows," *Comput. Methods Appl. Mech. Eng.*, vol. 3, pp. 269–289, 1974.

[69] W. Chen, Y. Ren, and L. Zhang, "Large Eddy Simulation on the Two-Phase Flow in a Water Model of Continuous Casting Strand with Gas Injection," *Steel Res. Int.*, vol. 1800287, pp. 1–12, 2018.

[70] T. Vu, C. Nguyen, and D. Khanh, "Direct Numerical Study of a Molten Metal Drop Solidifying on a Cold Plate with Different Wettability," *Metals (Basel).*, vol. 8, no. 1, p. 47, 2018.

[71] L. C. Hibbeler, R. Liu, and B. G. Thomas, "Review of Mold Flux Entrainment Mechanisms and Model Investigation of Entrainment by Shear-Layer Instability Meniscus Freezing and Hook Formation Another mechanism for the entrainment of slag and," *InSteelCon*, no. July, pp. 1–10, 2011.

[72] C. Kratzsch, K. Timmel, S. Eckert, and R. Schwarze, "URANS Simulation of Continuous Casting Mold Flow: Assessment of Revised Turbulence Models," *steel Res. Int.*, vol. 86, no. 4, pp. 400–410, Apr. 2015.

[73] L. C. Hibbeler and B. G. Thomas, "Mold slag entrainment mechanisms in continuous casting molds," *Iron Steel Technol.*, vol. 10, no. 10, pp. 121–136, 2013.

[74] *ANSYS FLUENT Theory Guide*, 18.2., no. August. Canonsburg, PA: ANSYS Inc. USA, 2017.

[75] M. I. H. Siddiqui and P. K. Jha, "Numerical Analysis of Heat Transfer and Flow Behaviour inside Different Shapes of Multi-Strand Continuous Casting Tundish," in *2nd National Conference on Advances in Heat Transfer and Fluid Dynamics, AMU, Aligarh, India*, 2013, pp. 65–72.

[76] M. Alam and M. I. H. Siddiqui, "CFD simulation of melt and inclusion motion in a mold under the influence of electromagnetic force," *VW Appl. Sci.*, vol. 1, no. 1, pp. 7–14, 2019.

[77] M. I. H. Siddiqui and P. K. Jha, "Multi-phase analysis of steel-air-slag system during ladle change-over process in CC tundish steelmaking process," in *Asia Steel Conference*.

[78] M. I. H. Siddiqui *et al.*, "Physical Investigations of Grade Mixing Phenomenon in Delta Shape Steel-making Tundish," *Int. Conf. CETCME, NIET, Noida, India*, vol. 2, no. 13, pp. 94–98, 2015.

[79] M. V. More, S. K. Saha, V. Marje, and G. Balachandran, "Numerical model of liquid metal flow in steel making tundish with flow modifiers," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 191, no. 1, 2017.

[80] R. Chaudhary, C. Ji, B. G. Thomas, and S. P. Vanka, "Transient turbulent flow in a liquid-metal model of continuous casting, including comparison of six different methods," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 42, no. 5, pp. 987–1007, 2011.

[81] J. S. Ha, J. R. Cho, B. Y. Lee, and M. Y. Ha, "Numerical analysis of secondary cooling and bulging in the continuous casting of slabs," *J. Mater. Process. Technol.*, vol. 113, no. 1–3, pp. 257–261, 2001.

[82] J. R. Boehmer, F. N. Fett, and G. Funk, "Analysis of high-temperature behaviour of solidified material within a continuous casting machine," *Comput. Struct.*, vol. 47, no. 4–5, pp. 683–698, 1993.

[83] A. M, E. H, and S. A., "Mathematical modeling of heat transfer for steel continuous casting process," *Int J ISSI*, vol. 3, no. 2, pp. 7–16, 2006.

[84] Q. Wang, Z. He, B. Li, and F. Tsukihashi, "A General Coupled Mathematical Model of Electromagnetic Phenomena, Two-Phase Flow, and Heat Transfer in Electroslag Remelting Process Including Conducting in the Mold," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 45, no. 6, pp. 2425–2441, 2014.

[85] A. C. Kheirabadi and D. Groulx, "the Effect of the Mushy-Zone Constant on Simulated Phase Change Heat Transfer," *Proceeding Proc. CHT-15. 6th Int. Symp. Adv. Comput. HEAT Transf. , May 25-29, 2015, Rutgers Univ. New Brunswick, NJ, USA*, no. May, p. 22, 2015.

[86] A. Maurya and P. K. Jha, "Influence of electromagnetic stirrer position on fluid flow and solidification in continuous casting mold," *Appl. Math. Model.*, vol. 48, pp. 736–748, 2017.

[87] K. Härkki and J. Miettinen, "Mathematical modeling of copper and brass upcasting," *Metall. Mater. Trans. B*, vol. 30, no. 1, pp. 75–98, Feb. 1999.

[88] M. L. S. Zappulla, L. C. Hibbeler, and B. G. Thomas, "Effect of Grade on Thermal–Mechanical Behavior of Steel During Initial Solidification," *Metall. Mater. Trans. A Phys. Metall. Mater. Sci.*, vol. 48, no. 8, pp. 3777–3793, Aug. 2017.

[89] C. A. M. Pinheiro, I. V. Samarasekera, J. K. Brimacomb, and B. N. Walker, "Mould heat transfer and continuously cast billet quality with mould flux lubrication Part 1 Mould heat transfer," *Ironmak. Steelmak.*, vol. 27, no. 1, pp. 37–54, Feb. 2003.

[90] S. Louhenkilpi, M. Mäkinen, S. Vapalahti, T. Räisänen, and J. Laine, "3D steady state and transient simulation tools for heat transfer and

solidification in continuous casting," *Mater. Sci. Eng. A,* vol. 413–414, pp. 135–138, 2005.

[91] A. A. Ivanova, "Calculation of Phase-Change Boundary Position in Continuous Casting," *Arch. Foundry Eng.*, vol. 13, no. 4, pp. 57–62, 2013.

[92] D. Zhang, S. Lei, S. Zeng, and H. Shen, "Thermo-mechanical Modeling in Continuous Slab Casting Mould and Its Application," *ISIJ Int.*, vol. 54, no. 2, pp. 336–341, 2014.

[93] Ambrish Maurya and Pradeep Kumar Jha, "Effect of Casting Speed on Continuous Casting of Steel Slab," *Int. J. Mech. Eng. Robot. Res.*, vol. 1, no. 1, pp. 13–21, 2014.

[94] P. T. Hietanen, S. Louhenkilpi, and S. Yu, "Investigation of Solidification, Heat Transfer and Fluid Flow in Continuous Casting of Steel Using an Advanced Modeling Approach," *Steel Res. Int.*, vol. 88, no. 7, pp. 1–13, 2017.

[95] L. C. Hibbeler, M. M. Chin See, J. Iwasaki, K. E. Swartz, R. J. O'Malley, and B. G. Thomas, "A reduced-order model of mould heat transfer in the continuous casting of steel," *Appl. Math. Model.*, vol. 40, no. 19–20, pp. 8530–8551, 2016.

[96] M. Vynnycky and S. Saleem, "On the explicit resolution of the mushy zone in the modelling of the continuous casting of alloys," *Appl. Math. Model.*, vol. 50, pp. 544–568, 2017.

[97] Y. Yin, J. Zhang, Q. Dong, and Q. H. Zhou, "Effects of electromagnetic stirring on fluid flow and temperature distribution in billet continuous casting mould and solidification structure of 55SiCr," *Ironmak. Steelmak.*, vol. 46, no. 9, pp. 855–864, 2019.

[98] O. Richter, J. Turnow, N. Kornev, and E. Hassel, "Numerical simulation of casting processes: coupled mould filling and solidification using VOF and enthalpy-porosity method," *Heat Mass Transf. und Stoffuebertragung*, vol. 53, no. 6, pp. 1957–1969, 2017.

[99] T. Oconnor and D. JA, "Modeling the thin-slab continuous-casting mold," *Metall. Mater. Trans. B*, vol. 25B, no. 6, p. 443, 1994.

[100] I. V. Samarasekera, D. L. Anderson, and J. K. Brimacombe, "The thermal distortion of continuous-casting billet molds," *Metall. Trans. B*, vol. 13, no. 1, pp. 91–104, 1982.

[101] H. A. N. K. Hwan, J. Yoon, and J. L. E. E. H. Nam, "Coupled Analysis of Fluid Flow , Heat Transfer and Stress Continuous Round Billet Oasting Thermal Analysis in Mold," *ISIJ Int.*, vol. 39, no. 5, pp. 435–444, 1999.

[102] V. D. Fachinotti, S. Le Corre, N. Triolet, M. Bobadilla, and M. Bellet, "Two-phase thermo-mechanical and macrosegregation modelling of binary alloys solidification with emphasis on the secondary cooling stage of steel slab continuous casting processes," *Int. J. Numer. Methods Eng.*, vol. 67, no. 10, pp. 1341–1384, Sep. 2006.

[103] Y. Hebi, Y. Man, Z. Huiying, and F. Dacheng, "3D Stress Model with Friction in and of Mould for Round Billet Continuous Casting," *ISIJ Int.*, vol. 46, no. 4, pp. 546–552, 2006.

[104] J. E. Lee, T. J. Yeo, O. H. Kyu Hwan, J. K. Yoon, and U. S. Yoon, "Prediction of cracks in continuously cast steel beam blank through fully coupled analysis of fluid flow, heat transfer, and deformation behavior of a solidifying shell," *Metall. Mater. Trans. A Phys. Metall. Mater. Sci.*, vol. 31, no. 1, pp. 225–237, 2000.

[105] X. Liu and M. Zhu, "Finite Element Analysis of Thermal and Mechanical Behavior in a Slab Continuous Casting Mold," *ISIJ Int.*, vol. 46, no. 11, pp. 1652–1659, 2006.

[106] Y. jun Li, H. Li, P. Lan, H. yan Tang, and J. quan Zhang, "Thermo-elasto-visco-plastic finite element analysis on formation and propagation of of-corner subsurface cracks in bloom continuous casting," *J. Iron Steel Res. Int.*, vol. 24, no. 11, pp. 1159–1168, 2017.

[107] M. R. Ridolfi, "Hot tearing modeling: A microstructural approach applied to steel solidification," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 45, no. 4, pp. 1425–1438, 2014.

[108] S. Saleem, M. Vynnycky, and H. Fredriksson, "The Influence of Peritectic Reaction/Transformation on Crack Susceptibility in the Continuous Casting of Steels," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 48, no. 3, pp. 1625–1635, 2017.

[109] J. M. Risso, A. E. Huespe, and A. Cardona, "Thermal stress evaluation in the steel continuous casting process," *Int. J. Numer. Methods Eng.*, vol. 65, no. 9, pp. 1355–1377, 2006.

[110] Z. Chen, H. Olia, B. Petrus, M. Rembold, J. Bentsman, and B. G. Thomas, "Dynamic Modeling of Unsteady Bulging in Continuous Casting of Steel," in *Materials Processing Fundamentals*, 2019, pp. 23–35.

[111] A. G. Weinberg, Brimacombe, and J. K. F., "Mathematical analysis of stress in continuous casting of steel," *Ironmak. Steelmak.*, vol. 3, no. 1, pp. 38–47, 1976.

[112] J. E. Kelly, K. P. Michalek, T. G. O'Connor, B. G. Thomas, and J. A. Dantzig, "Initial development of thermal and stress fields in continuously cast steel billets," *Metall. Trans. A, Phys. Metall. Mater. Sci.*, vol. 19 A, no. 10, pp. 2589–2602, Oct. 1988.

[113] J. O. Kristiansson, "Thermomechanical behavior of the solidifying shell within continuous-casting billet molds-a numerical approach," *J. Therm. Stress.*, vol. 7, no. 3–4, pp. 209–226, Jan. 1984.

[114] M. Y. Zhu, Z. Z. Cai, and H. Q. Yu, "Multiphase Flow and Thermo-Mechanical Behaviors of Solidifying Shell in Continuous Casting Mold," *J. Iron Steel Res. Int.*, vol. 20,

no. 3, pp. 6–17, 2013.

[115] G. Funk, J. R. Boehmer, and F. N. Fett, “A coupled FDM/FEM model for the continuous casting process,” *Int. J. Comput. Appl. Technol.*, vol. 7, no. 3–6, 1994.

[116] M. Samonds and J. Z. Zhu, “Coupled Thermal-fluids-stress Analysis of Castings,” in *Proc. 9 th Int. Conf. on Modeling of Casting*, 2000.

[117] F. Pascon, S. Cescotto, and A. M. Habraken, “A 2.5D finite element model for bending and straightening in continuous casting of steel slabs,” *Int. J. Numer. Methods Eng.*, vol. 68, no. 1, pp. 125–149, Oct. 2006.

[118] E. Takeuchi, “Applying MHD technology to the continuous casting of steel slab,” *Jom*, vol. 47, no. 5, pp. 42–45, 1995.

[119] R. Chaudhary, B. G. Thomas, and S. P. Vanka, “Effect of electromagnetic ruler braking (embr) on transient turbulent flow in continuous slab casting using large eddy simulations,” *Metall. Mater. Trans. B*, vol. 43, no. 3, pp. 532–553, Feb. 2012.

[120] K. Jin, S. P. Vanka, and B. G. Thomas, “Large Eddy Simulations of the Effects of EMBr and SEN Submergence Depth on Turbulent Flow in the Mold Region of a Steel Caster,” *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 48, no. 1, pp. 1–17, 2016.

[121] S.-M. Cho, B. G. Thomas, and S.-H. Kim, “Transient Two-Phase Flow in Slide-Gate Nozzle and Mold of Continuous Steel Slab Casting with and without Double-Ruler Electro-Magnetic Braking,” *Metall. Mater. Trans. B*, vol. 47, no. 5, pp. 3080–3098, 2016.

[122] H. K. Moffatt, “Electromagnetic stirring,” *Phys. Fluids A Fluid Dyn.*, vol. 3, no. 5, pp. 1336–1343, May 1991.

[123] M. Javurek, M. Barna, P. Gittler, K. Rockenschaub, and M. Lechner, “Flow modelling in continuous casting of round bloom strands with electromagnetic stirring,” *Steel Res. Int.*, vol. 79, no. August, pp. 617–626, 2008.

[124] B. A. Sivak *et al.*, “MHD processes in the electromagnetic stirring of liquid metal in continuous section and bloom casters,” *Metallurgist*, vol. 53, no. 7–8, pp. 469–481, Jul. 2009.

[125] A. Maurya and P. K. Jha, “Study of fluid flow and solidification in billet caster continuous casting mold with electromagnetic stirring,” *Arch. Metall. Mater.*, vol. 63, no. 1, pp. 413–424, 2018.

[126] F. C. Chang, J. R. Hull, and L. Beitelman, “Simulation of flow control in the meniscus of a continuous casting mold with opposing alternating current magnetic fields,” *Metall. Mater. Trans. B*, vol. 35, no. December, pp. 1129–1137, 2004.

[127] A. Cramer, S. Eckert, V. Galindo, G. Gerbeth, B. Willers, and W. Witke, “Liquid metal model experiments on casting and solidification processes,” *J. Mater. Sci.*, vol. 39, no. 24, pp. 7285–7294, 2004.

[128] S. Eckert, S. Gerbeth, S. Räbiger, B. Willers, and C. Zhang, “Experimental modeling using low melting point metallic melts: Relevance for metallurgical engineering.,” *Steel Res. Int.*, vol. 78, pp. 419–425, 2007.

[129] M. Cho, E. Park, and S. Kim, “Shield for improving wavy meniscus in the billet continuous casting mold with electromagnetic stirring,” *ISIJ Int.*, vol. 50, no. 8, pp. 1180–1184, 2010.

[130] N. Vogl, H.-J. Odenthal, M. Reifferscheid, and J. Schlüter, “Advanced Simulation of Mold Flow Influenced by Static and Moving Magnetic Fields,” in *The Iron & Steel Technology Conference and Exposition*, 2013, vol. 49, pp. 1519–1529.

[131] J. Zeng, W. Chen, S. Zhang, Y. Li, and Q. Wang, “Development and Application of Final Permanent Magnet Stirring during Continuous Casting of High Carbon Rectangular Billet,” *ISIJ Int.*, vol. 55, no. 10, pp. 2142–2149, 2015.

[132] H. An, Y. Bao, M. Wang, and L. Zhao, “Numerical Analysis of Coupled Turbulent Flow and Macroscopic Solidification in a Billet Continuous Casting Mould with Electromagnetic Stirring,” in *TMS 2017 146th Annual Meeting & Exhibition Supplemental Proceedings*, 2017, p. 551.

[133] Y. Yin, J. Zhang, Q. Dong, and Q. H. Zhou, “Numerical Simulation of Fluid Flow, Heat Transfer, Species Transfer, and Solidification in Billet Continuous Casting Mold with M-EMS,” *Ironmak. Steelmak.*, vol. 46, no. 9, pp. 855–864, 2019.

[134] D. Schurmann, B. Willers, G. Hackl, Y. Tang, and S. Eckert, “Experimental Study of the Mold Flow Induced by a Swirling Flow Nozzle and Electromagnetic Stirring for Continuous Casting of Round Blooms,” *Metall. Mater. Trans. B*, 2019.

[135] S. Koric and B. G. Thomas, “Efficient thermo-mechanical model for solidification processes,” *Int. J. Numer. Methods Eng.*, vol. 66, no. 12, pp. 1955–1989, Jun. 2006.

[136] S. Koric and B. G. Thomas, “Thermo-mechanical models of steel solidification based on two elastic visco-plastic constitutive laws,” *J. Mater. Process. Technol.*, vol. 197, no. 1–3, pp. 408–418, 2008.

[137] M. M. Yavuz, “The effects of electromagnetic brake on liquid steel flow in thin slab caster,” *Steel Res. Int.*, vol. 82, no. 7, pp. 809–818, 2011.

[138] L. L. Zhang, D. F. Chen, H. B. Chen, M. J. Long, and X. Xie, “Study on transport phenomena in the beam blank continuous casting mould coupling copper mould with molten steel,” *Ironmak. Steelmak.*, vol. 44, no. 3, pp. 193–201, 2017.

[139] S. H. Seyedein and M. Hasan, “A 3-D

Numerical Prediction of Turbulent Flow, Heat Transfer and Solidification in a Continuous Slab Caster for Steel," *Can. Metall. Q.*, vol. 37, no. 3–4, pp. 213–228, Oct. 1998.

[140] S. Seyedein, M. H.-I. J. of H. and M. Transfer, and undefined 1997, "A three-dimensional simulation of coupled turbulent flow and macroscopic solidification heat transfer for continuous slab casters," *Elsevier*.

[141] H. M. Si, C. Cho, and S. Y. Kwahk, "A hybrid method for casting process simulation by combining FDM and FEM with an efficient data conversion algorithm," *J. Mater. Process. Technol.*, vol. 133, no. 3, pp. 311–321, 2003.

[142] B. Shen, H. Shen, and B. Liu, "Instability of Fluid Flow and Level Fluctuation in Continuous Thin Slab Casting Mould," *ISIJ Int.*, vol. 47, no. 3, pp. 427–432, 2007.

[143] D. Li *et al.*, "Effects of Electromagnetic Swirling Flow in Submerged Entry Nozzle on Square Billet Continuous Casting of Steel Process," *ISIJ Int.*, vol. 53, no. 7, pp. 1187–1194, 2013.

[144] I. C. Ramos, R. D. Morales, S. Garcia-Hernandez, and A. Ceballos-Huerta, "Effects of Immersion Depth on Flow Turbulence of Liquid Steel in a Slab Mold Using a Nozzle with Upward Angle Rectangular Ports," *ISIJ Int.*, vol. 54, no. 8, pp. 1797–1806, 2014.

[145] F. Li, E. Wang, M. Feng, and Z. Li, "Simulation Research of Flow Field in Continuous Casting Mold with Vertical Electromagnetic Brake," *ISIJ Int.*, vol. 55, no. 4, pp. 814–820, 2015.

[146] Z. Liu, L. Li, and B. Li, "Large Eddy Simulation of Transient Flow and Inclusions Transport in Continuous Casting Mold under Different Electromagnetic Brakes," *Jom*, vol. 68, no. 8, pp. 2180–2190, 2016.

[147] B. Thomas, Q. Yuan, S. Sivaramakrishnan, T. Shi, S. F. Vanka, and M. B. Assar, "Comparison of four methods to evaluate fluid velocities in a continuous slab casting mold," *ISIJ Int.*, vol. 41, no. 10, pp. 1262–1271, 2001.

[148] O. Smirnov, O. Smyrnov, and S. Oskol, "Mathematical modelling of multiphase system in tundish of continuous casting machine : Theory and experiments," *V, Yevgen Smyrnov, Seppo Louhenkilpi, Oleg Smy*, vol. 52, no. 4, pp. 711–717, 2017.

[149] Q. Wang, B. Li, and F. Tsukihashi, "Modeling of a Thermo-Electromagneto-Hydrodynamic Problem in Continuous Casting Tundish with Channel Type Induction Heating," *ISIJ Int.*, vol. 54, no. 2, pp. 311–320, 2014.

[150] J. xin Song, Z. zhen Cai, F. yun Piao, and M. yong Zhu, "Heat Transfer and Deformation Behavior of Shell Solidification in Wide and Thick Slab Continuous Casting Mold," *J. Iron Steel Res. Int.*, vol. 21, no. SUPPL.1, pp. 1–9, 2014.

[151] B. G. Thomas, "Review on Modeling and Simulation of Continuous Casting," *Steel Res. Int.*, vol. 89, no. 1, pp. 1–21, 2018.

[152] B. G. Thomas and J. Sengupta, "The visualization of defect formation during casting processes," *JOM*, vol. 58, no. 12. pp. 16–18, 2006.

[153] Z. Liu, B. Li, M. Jiang, and F. Tsukihashi, "Modeling of Transient Two-Phase Flow in a Continuous Casting Mold Using Euler-Euler Large Eddy Simulation Scheme," *ISIJ Int.*, vol. 53, no. 3, pp. 484–492, 2013.

[154] C. Li and B. G. Thomas, "Thermomechanical finite-element model of shell behavior in continuous casting of steel," *Metall. Mater. Trans. B Process Metall. Mater. Process. Sci.*, vol. 35, no. 6, pp. 1151–1172, Dec. 2004.

[155] T. Telejko, Z. Malinowski, and M. Rywotycki, "Analysis of heat transfer and fluid flow in continuous steel casting," *Arch. Metall. Mater.*, vol. 54, no. 3, pp. 837–844, 2009.

[156] S. Chakraborty, S. Ganguly, and P. Talukdar, "Determination of optimal taper in continuous casting billet mould using thermo-mechanical models of mould and billet," *J. Mater. Process. Technol.*, vol. 270, no. December 2018, pp. 132–141, 2019.

[157] S. Mosayebidorcheh and M. Gorji-Bandpy, "Local and averaged-area analysis of steel slab heat transfer and phase change in continuous casting process," *Appl. Therm. Eng.*, 2017.

[158] A. Maurya and P. K. Jha, "Influence of electromagnetic stirrer position on fluid flow and solidification in continuous casting mold," *Appl. Math. Model.*, vol. 48, no. 8, pp. 736–748, 2017.